Physical Models of Offroad Simulator

This document describes the simulator as a coupled mechanical, pneumatic, thermal, and numerical system. It is intended for readers familiar with rigid-body dynamics, analytical mechanics, tire modeling, and control theory. Equations describe the implemented model, including empirical closures and numerical regularization; they do not establish experimental validation for a particular production vehicle.

For installation, operating procedures, input mappings, and the settings panel, see the accompanying "User manual". Values identified as defaults are initial model parameters. Saved settings can change them. All equations use SI units unless another unit is explicitly shown.

The contact and terrain sections follow the implementation documented for this build, including geometric preparation reuse and parallel wheel-contact evaluation. These execution techniques retain the physical equations and numerical integration rules described below.

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1. Model architecture and scope

The vehicle is a hybrid dynamical system. Its principal mechanical subsystem consists of one free chassis rigid body, two solid axles with two reduced coordinates each, four wheel spin states, a lumped engine rotor, and steering and drivetrain actuator states. The chassis is integrated by Unreal Engine's Chaos rigid-body solver. A separate vehicle kernel computes suspension reactions, elastic tire contact, traction, drivetrain torques, and braking.

The pneumatic reaction control system (RCS) adds eight physical thrust forces at body-mounted nozzle locations. Stored gas changes sprung mass, center of mass, and inertia. Optional assistance computes valve requests; it uses the same pressure, flow, gas inventory, and force application as manual flight.

Subsystem	Representation	Main coupling
Chassis	Free six-degree-of-freedom rigid body	Gravity, suspension reactions, body collisions, drag, rotor reactions, RCS
Each solid axle	Heave and roll coordinates with geometric linkage closure	Tire wrench, springs, dampers, stops, anti-roll bar, moving chassis
Each tire carcass	Rounded, weighted contact volume and elastic potential	Terrain/support geometry, normal load, pressure-dependent stiffness
Each wheel spin	One angular velocity and angle	Drive torque, tire longitudinal force, brakes, rolling resistance
Engine and transmission	Lumped rotor, torque map, clutch/converter and discrete gears	Throttle, shaft speed, wheel torque
Tire and brake thermal states	Lumped temperature and empirical wear	Slip work, rolling loss, brake work, cooling
RCS	Ideal isothermal storage, valve dynamics, finite shared flow	Thrust, gas consumption, compressor energy, variable mass
Environment and presentation	Prescribed geometry, optical volumes, procedural audio and feedback	Contact materials and visual or sensory observables

This is not a finite-element vehicle, a flexible multibody suspension, a combustion-cycle solver, a tire structural solver, or a compressible-flow calculation. Its detailed contact geometry should not be interpreted as detailed rubber constitutive mechanics. Conversely, the reduced suspension does include linkage-induced lateral and longitudinal movement, inertial coupling, and body reactions; it is more than four independent vertical support springs.

The distinction between a physical closure and an assistance law matters. ABS, hill assistance, steering smoothing, RCS damping, and automatic roll are controllers acting on the modeled plant. Tire wear, brake fade, aerodynamic drag area, and engine torque factors are empirical relations. Fog, dust, visible exhaust, and synthesized sound are presentation models.

2. Coordinates, units, and state

2.1 Frames and signs

The world vertical points upward. Gravity is

$$\mathbf{g} = (0, 0, -9.81)\ \mathrm{m\,s^{-2}}.$$

Vehicle-local axes are forward x, right y, and up z. Let $\mathbf{R} = [\mathbf{f}\ \mathbf{r}\ \mathbf{u}]$ map a local vector into the world frame. The chassis reference origin $\mathbf{x}_O$ and its center of mass $\mathbf{x}_C$ are distinct:

$$\mathbf{x}_C = \mathbf{x}_O + \mathbf{R}\mathbf{c}.$$

Here $\mathbf{c}$ is the local sprung center of mass. A local point $\mathbf{a}$ has world position $\mathbf{x}_O + \mathbf{R}\mathbf{a}$. With chassis center-of-mass velocity $\mathbf{V}$ and world angular velocity $\boldsymbol{\Omega}$, its rigid-body velocity is

$$\mathbf{v}(\mathbf{a}) = \mathbf{V} + \boldsymbol{\Omega} \times \mathbf{R}(\mathbf{a} - \mathbf{c}).$$

Positive axle roll raises its right end in local coordinates. Wheel rolling speed is signed; the longitudinal slip definition supports forward and reverse motion. RCS lever arms are measured from $\mathbf{c}$, rather than from the mesh origin.

2.2 Units at the engine boundary

The vehicle kernel uses meters, kilograms, seconds, radians, newtons, and newton-meters. Unreal positions and linear velocities use centimeters. At the solver interface, a force in newtons is multiplied by 100 and a torque in newton-meters by 10,000. Inertia in kg m² is multiplied by 10,000 when assigned to the centimeter-based solver. Angular velocity remains in rad/s.

Displayed RPM, degrees, Celsius, bar, kW, and kWh are converted explicitly where used. Tire pressure is gauge pressure. RCS storage pressure is treated as absolute pressure in the ideal-gas and compressor calculations. These pressure conventions are different.

2.3 State and algebraic quantities

The independent or persistent states include chassis pose and velocity; axle heave, roll, and their rates; wheel angle and angular velocity; engine angular velocity; tire force relaxation states; tire temperature and wear; brake temperature, pressure and ABS multipliers; steering angle and rate; gear and shift timers; RCS gas and battery inventories and valve states; and assistance-controller memories.

Contact locations, normal loads, slip, instantaneous mass properties, linkage closure, and differential torque allocations are computed from those states. A displayed contact point is a representative point for an extended contact calculation. It does not identify a single physical bristle or a resolved stress maximum.

3. Time integration and coupling

3.1 Three time scales

The project requests asynchronous rigid-body physics with a fixed outer interval

$$\Delta t = 1/240 \text{ s}.$$

The mechanical kernel divides each accepted callback into

$$n_m = \max(3, \lceil \Delta t / 0.0014 \rceil), \qquad h_m = \Delta t / n_m.$$

At the nominal outer interval this is three mechanical substeps, approximately 720 Hz. The RCS uses $n_r = \lceil 960 \Delta t \rceil$, giving four pneumatic substeps, approximately 960 Hz. These are internal state and force updates. They are not separate full Chaos chassis integration steps.

During the mechanical substeps, the sampled chassis orientation and velocities are held fixed. Its reference position is predicted by linear translation from the outer snapshot. Linkage coordinates, wheel spin, engine speed, contact forces, and thermal states evolve internally. The kernel returns the arithmetic mean of its substep forces and torques. The RCS likewise returns its mean wrench. Chaos then integrates the chassis with those loads, its own gravity, and its own collision constraints.

Consequently, suspension/tire/chassis coupling is partitioned rather than solved as a single implicit global system. Very stiff settings, fast impacts, rapid rotation, and sharply changing support geometry expose this distinction.

3.2 Moving-base acceleration estimate

The axle equations need chassis translational and angular acceleration. These are obtained by differentiating successive outer-step velocities, bounding the estimated vector norms to $8g$ and $60\ \mathrm{rad\,s^{-2}}$, and low-pass filtering with a 0.035 s time constant:

$$\hat{\mathbf{a}}^{+} = \hat{\mathbf{a}} + \left(1 - e^{-\Delta t/0.035}\right)\left(\mathbf{a}_{\mathrm{bounded}} - \hat{\mathbf{a}}\right).$$

The angular estimate uses the same form. These bounds condition the moving-base terms in the reduced axle solver. They do not cap the actual chassis acceleration, impact force, speed, angular velocity, or RCS thrust.

3.3 Numerical transitions

The mechanical kernel rejects nonpositive intervals and intervals above 0.025 s. The RCS bounds its processed interval at 0.1 s before subdivision. These protections do not make a stalled or overloaded execution equivalent to a correctly timed trajectory.

Axle travel is projected onto its admissible end-position range after integration. Wheel and clutch solves use bounded implicit relations. Steering and valves use exponential updates for constant requests. Gear engagement, locks, range selection, ABS thresholds, reserve entry, contact creation/loss, and recovery introduce discrete transitions.

A fixed requested physics interval improves repeatability. It does not guarantee bitwise determinism across machines, solver scheduling, floating-point implementations, or collision histories.

3.4 Contact preparation and parallel scheduling

Within each mechanical substep, wheel centers and axes are prepared in the original wheel order. Solver-time scene queries and supported-solid extraction run serially before dispatch; their results are copied into private per-wheel input storage. On an eligible threaded configuration, the four pure solid-contact jobs can run concurrently, using at most three scheduler workers plus the calling physics thread. Supported scene solids do not by themselves disable this batch. Each job has a distinct wheel cache and output; it does not mutate the shared vehicle state or apply reactions to Chaos.

The batch joins before the consuming wheel loop continues. Tire, traction and suspension updates, accumulated vehicle loads, and support-reaction callbacks retain their original wheel order. The caller's floating-point controls are installed for each job, worker environments are restored, and resulting exception flags are combined on return. Unsupported or compatibility contacts, unsafe shared-cache ownership, unavailable threading, or an occupied batch retain the compatible serial route. These are execution choices within the existing substeps; they do not reduce the requested physics frequency, postpone contact to a later frame, or reuse a previous contact wrench. They do not establish cross-machine bitwise determinism.

4. Mass, center of mass, and inertia

4.1 Sprung mass assembly

Let each fixed body-local mass component have mass m_j, center $\mathbf{c}_j$, and centroidal inertia $\mathbf{I}_j$. The assembled sprung properties are

$$M = \sum_j m_j, \qquad \mathbf{c} = \frac{\sum_j m_j \mathbf{c}_j}{M},$$

$$\mathbf{I}_C = \sum_j \left[\mathbf{I}_j + m_j \left((\mathbf{d}_j \cdot \mathbf{d}_j)\mathbf{1} - \mathbf{d}_j \mathbf{d}_j^{\mathsf{T}}\right)\right], \qquad \mathbf{d}_j = \mathbf{c}_j - \mathbf{c}.$$

The complete symmetric tensor includes products of inertia. It is diagonalized by Jacobi rotations to supply Chaos with principal moments and their orientation. The vehicle and RCS calculations retain the full tensor where specified.

The base sprung mass is 1,850 kg, at $(0, 0, -0.22)$ m by default. Its centroidal tensor is

$$\mathbf{I}_{\text{base}} = \frac{M_{\text{base}}}{1850}\,\mathrm{diag}(950, 3100, 3400)\ \mathrm{kg\,m^2}.$$

Additional components are equivalent rectangular volumes. For dimensions (a, b, c),

$$\mathbf{I}_{\text{box}} = \frac{m}{12}\,\mathrm{diag}(b^2 + c^2, a^2 + c^2, a^2 + b^2).$$

Component	Center x, y, z (m)	Equivalent dimensions (m)
Driver	$-0.10, -0.36, 0.55$	$0.45, 0.45, 1.00$
Passenger	$-0.10, 0.36, 0.55$	$0.45, 0.45, 1.00$
Rear cargo	$-1.20, y_{\text{cargo}}, 0.50$	$0.80, 1.00, 0.50$
Roof load	$-0.65, 0, 1.30$	$0.80, 1.10, 0.35$
Fuel load	$-1.15, 0, -0.25$	$0.50, 0.60, 0.25$
RCS equipment and stored gas	$0, 0, -0.10$	$1.20, 0.80, 0.35$

These are mechanical mass distributions, not occupants or cargo bodies that move independently. Fuel is a fixed added load; engine operation does not burn it. There is no fuel slosh, cargo motion, occupant articulation, or flexible roof load.

4.2 Unsprung mass and static distribution

Each axle mass is $m_a = 2m_u$, where the default $m_u = 47.5$ kg is the per-wheel unsprung mass parameter. Thus the two axle assemblies add 190 kg to the complete vehicle, outside the Chaos sprung body. Their inertia and gravity enter through the reduced axle equations and reactions.

The initial sprung front/rear load split uses

$$M_F = M\left(\tfrac{1}{2} + c_x/L\right), \qquad M_R = M\left(\tfrac{1}{2} - c_x/L\right),$$

where L is wheelbase. It is an initialization relation. Actual contact loads subsequently follow the full dynamic solution, including acceleration, slope, CG height, body attitude, anti-roll coupling, and gas consumption.

Wheel spin inertia is modeled separately from the axle's translational mass and housing inertia. The initial axle inertia in its own forward/right/up frame is approximately

$$\mathbf{I}_a = \operatorname{diag}\left(J_{\text{roll}},\, 5.4\, m_a/95,\, J_{\text{roll}}\right).$$

The default $J_{\text{roll}} = 55\ \text{kg}\,\text{m}^2$ is independent for front and rear.

4.3 Changing mass

User edits to structural parameters are accepted under stopped-state conditions. They are externally imposed changes to the physical system and can change its stored energy. RCS consumption and recharge change gas mass during simulation; physical-thread body mass, inverse mass, local CG, and principal inertia are updated without deliberately resetting motion.

RCS substeps use one mass-property snapshot per outer RCS call. Gas inventory changes within that call, followed by the body-property update. This creates a finite-step lag in mass/thrust coupling; it is not a continuously solved variable-mass multibody equation.

5. Chassis dynamics and rigid collisions

For fixed instantaneous mass properties, the conceptual chassis equations are

$$M\dot{\mathbf{V}} = M\mathbf{g} + \sum \mathbf{F}, \qquad \mathbf{I}_C\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{I}_C\boldsymbol{\omega}) = \sum \boldsymbol{\tau}_C$$

in the appropriate world/body frames. Chaos owns pose integration and general body collision impulses. The separate kernel supplies suspension, tire-derived axle reactions, aerodynamic drag, modeled rotor reactions, residual damping, and RCS loads.

The collision representation is a welded set of rigid protective shapes around the chassis, body, and nominal wheel cores. The visual exterior is not an elastic crash mesh. These shapes do not add independent mass to the analytically assembled body. Protective wheel cores remain at nominal body-mounted geometry; they are not four separate suspended Chaos wheel bodies.

The assigned body collision material has dynamic friction 0.65, static friction 0.8, and zero restitution, with minimum restitution combination. These coefficients belong to general rigid contacts. Tire rolling traction uses the separate contact and surface laws below. Zero restitution does not eliminate all rebound: tire and suspension elastic energy can still produce it.

Continuous collision detection is enabled for the chassis. The project sets p.Chaos.Solver.Collision.MaxPushOutVelocity to 20 cm/s to bound numerical recovery of residual penetration in ordinary rigid contacts. This solver-wide setting also affects dynamic props. It does not clamp physical vehicle or impact velocity, and is distinct from the body initial-overlap depenetration setting of 150 cm/s. Protective geometry and solver iteration counts remain unchanged. Collision stabilization, finite iterations, and travel projection are numerical operations with possible energy error. There is no damage, plastic crushing, chassis torsional flex, tire puncture, or automatic righting torque.

For a tire on a moving rigid support, support-point velocity comes from the physics-thread rigid body:

$$\mathbf{v}_s(\mathbf{x}) = \mathbf{V}_s + \boldsymbol{\Omega}_s \times (\mathbf{x} - \mathbf{x}_{Cs}).$$

Dynamic supports receive the opposite contact work-conjugate force and moment contributions. Fixed terrain receives no integrated reaction motion; it represents an effectively infinite support. Render-thread interpolation is not used to supply physical support velocity.

6. Suspension and solid-axle dynamics

6.1 Reduced geometry and constraints

Each axle has generalized coordinates

$$\mathbf{q} = (h, \phi)^{\mathsf{T}},$$

with heave h and roll ϕ. Relative axle pitch is not an independent coordinate. Longitudinal displacement x, lateral displacement y, and yaw ψ are determined by two fixed-length longitudinal arms and a Panhard rod.

The axle center is $(x, y, -0.16 - \ell_0 + h)$. Its forward axis is $(\cos\psi, \sin\psi, 0)$; its right axis is $(-\sin\psi\cos\phi, \cos\psi\cos\phi, \sin\phi)$. Their cross product gives its up axis. Wheel-center separation is always the configured track.

Let the three squared-length constraints be $\mathbf{f}(\mathbf{u}, \mathbf{q}) = 0$, with $\mathbf{u} = (x, y, \psi)$. A Newton solve, limited to 12 iterations, closes them at each geometry evaluation. Increments are bounded to 0.2 m in x, y and 0.15 rad in yaw. The linkage Jacobian gives

$$\frac{\partial \mathbf{u}}{\partial q_i} = -\left(\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right)^{-1} \frac{\partial \mathbf{f}}{\partial q_i}.$$

This provides axle-center translational Jacobians $\mathbf{J}_i$ and orientation/angular Jacobians $\mathbf{A}_i$. Velocity bias terms use centered directional differentiation of these Jacobians along $\dot{\mathbf{q}}$, with a 0.0005 s differentiation interval.

The arm length defaults to 0.95 m for each axle. The Panhard length scales as $1.45\,t/1.68$ m with track t. Its asymmetric mounting produces lateral axle motion and yaw during articulation. Constraint closure error is a diagnostic quantity, not an additional elastic bushing.

6.2 Springs and dampers

Spring seats are located at lateral offsets $\pm t/3$, rather than at the wheel centers $\pm t/2$. Let the current seat-to-body distance be ℓ_i and its corresponding reference distance ℓ_{i0}. Compression is

$$c_i = \ell_{i0} - \ell_i.$$

Its rate is evaluated by projecting the seat's relative velocity along the seat-to-body direction. Each side develops a spring/damper scalar

$$F_i = k c_i + d_{\pm}\, v_{\text{eff}}(\dot{c}_i),$$

$$v_{\text{eff}}(v) = \text{clip}(v, -0.45, 0.45) + 0.25\,[v - \text{clip}(v, -0.45, 0.45)].$$

The compression or rebound damping coefficient is selected by the sign of $\dot{c}_i$. Above 0.45 m/s, the incremental damping slope is one quarter of its low-speed value. The force is applied along the actual spring-seat line, with its moment about the axle center.

Defaults per side are $k = 43{,}000$ N/m, compression damping $2{,}800$ N s/m, and rebound damping $4{,}200$ N s/m. Front and rear parameters are independently configurable.

6.3 Anti-roll bars and travel stops

An anti-roll bar transfers equal and opposite seat forces proportional to differential compression:

$$F_{\text{bar}} = k_{\text{bar}}\, b (c_R - c_L), \qquad U_{\text{bar}} = \tfrac{1}{2} k_{\text{bar}}\, b (c_R - c_L)^2.$$

Here $b \in [0, 1]$ is connection engagement, ramped over approximately 0.35 s when switched, and $k_{\text{bar}} = 9{,}000$ N/m by default. Changing b at nonzero articulation changes potential energy through an external configuration action.

Travel is evaluated at the two axle ends:

$$c_{\text{end},\pm} = h \pm (t/2) \sin\phi.$$

With maximum compression C and droop D, define

$$\beta = \max[0, c_{\text{end}} - (C - 0.035)], \qquad \gamma = \min[0, c_{\text{end}} + D - 0.025].$$

The body-up stop-force scalar on the axle is

$$F_{\text{stop}} = -300000\, \beta^2 / 0.035 - 180000\, \gamma.$$

The associated potential is $100000\beta^3/0.035 + 90000\gamma^2$. These are progressive compression bump stops and linear extension stops, with their stated dimensional coefficients. After integration, both end positions are also projected into $[-D, C]$ and outward end velocities are removed. That final projection is an inelastic numerical travel limit, distinct from the continuous stop forces.

6.4 Reduced equations and body reaction

The generalized mass matrix is

$$\mathcal{M}_{ij} = m_a\, \mathbf{J}_i \cdot \mathbf{J}_j + \mathbf{A}_i \cdot \mathbf{I}_a \mathbf{A}_j.$$

In a common frame, the generalized force balance is

$$\mathcal{M}_{ij} \ddot{q}_j = \mathbf{J}_i \cdot (\mathbf{F}_a - m_a \mathbf{a}_{\text{base}}) + \mathbf{A}_i \cdot (\boldsymbol{\tau}_a - \mathbf{I}_a \boldsymbol{\alpha}_{\text{base}} - \boldsymbol{\omega}_a \times \mathbf{I}_a \boldsymbol{\omega}_a).$$

The force and moment sums include tire wrenches, axle gravity, spring/damper forces, anti-roll forces, and stops. The moving-base acceleration contains chassis acceleration, angular acceleration, centrifugal terms, Coriolis terms, and the Jacobian bias:

$$\mathbf{a}_{\text{base}} = \hat{\mathbf{a}}_C + \hat{\boldsymbol{\alpha}} \times \mathbf{r}_a + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_a) + 2\boldsymbol{\Omega} \times \mathbf{R}\mathbf{v}_{\text{rel}} + \mathbf{R}\mathbf{a}_{\text{bias}}.$$

The corresponding angular bias includes chassis angular acceleration, rotation of the relative angular velocity, and the angular Jacobian derivative. The two-by-two mass system is solved for $\ddot{\mathbf{q}}$, followed by semi-implicit velocity and coordinate updates. A determinant floor is a numerical safeguard, not an added mechanical compliance.

After travel projection, the actual rate change is used to reconstruct acceleration for reaction accounting. The chassis receives the axle momentum-balance reaction. In particular, its force contribution is equivalent to

$$\mathbf{F}_{C,a} = \sum_{\text{tires}} \mathbf{F}_i + m_a(\mathbf{g} - \mathbf{a}_a).$$

The torque balance also includes axle rotational inertia and gyroscopic terms, the axle-center offset, and tire moments. Tire loads are not then added a second time directly to the chassis. This is how unsprung gravity and inertia are transmitted while keeping axle mass outside the chassis rigid-body mass.

The model does not resolve arm mass distribution, ball-joint compliance, bushing hysteresis, housing torsion, bearing clearances, driveshaft geometry, or independent caster/camber/toe compliance.

7. Three-dimensional elastic tire contact

7.1 Contact geometry

Normal contact is based on the overlap of an oriented, rounded, weighted tire volume with supporting solids. The exhibition path retains the upper and lower boundaries of each solid, including tall step edges and bridge undersides. The compatibility height-sampler path is described separately below. It is not a single vertical ray spring or a three-point contact force model.

For tire radius R_t, width w, and unit axle direction $\mathbf{a}$, let

$$u = \mathbf{r} \cdot \mathbf{a}, \quad \rho = \|\mathbf{r} - u\mathbf{a}\|, \quad b = w/2,$$

$$R_c = (33.2/43.5)R_t, \qquad R_s = (10.3/43.5)R_t.$$

The support envelope is

$$\rho \le R_c + R_s\sqrt{1 - u^2/b^2}, \qquad |u| \le b.$$

Within it, the nonnegative integration weight is

$$q = \max(0, 1 - u^2/b^2), \quad s = \max(\rho - R_c, 0), \qquad \chi(\mathbf{r}) = q^2 \max(0, q - s^2/R_s^2).$$

Thus the center section has a broad cylindrical core and rounded shoulders. The weight vanishes smoothly at much of the envelope. It is a geometric contact weighting, not a rubber density or a material stress.

7.2 Weighted overlap and plane consistency

For the exhibition and supported scene collision geometry, the solid at each world-XY position is the union of the foundation half-space and closed intervals $[B_j(x,y), T_j(x,y)]$. The lower face is retained: a bridge is a finite slab rather than solid material extending downward from its driving surface. Overlapping intervals are merged before integration. If $C(h)$ denotes the weighted tire-column integral below height h, the overlap is

$$J_Q = \int_{\mathbb{R}^2} \sum_{[B,T] \in \text{union } \mathcal{S}(x,y)} \big[C(T) - C(B)\big]\, dx\, dy.$$

The generic calculation uses fixed world-XY cells at 0.04 m spacing. Cells cut by a solid boundary are clipped to the actual projected triangles and integrated with a degree-five, seven-point triangle rule. Immutable terrain cells wholly inside a continuous surface retain a midpoint rule, with refined cells around

real boundaries. External scene solids use the clipped triangle rule throughout, including moving supports. Each vertical tire column includes translation and orientation derivatives. A wheel-dependent height cap no longer decides whether an exhibition or supported scene solid exists; the complete tire volume can meet a tall side face or an underside before reaching the top.

A proved homogeneous immutable plane or slab uses its plane integrals directly. The proof checks coverage, body identity, upper and lower planes, material data, and rigid velocity consistency. The plane evaluator uses analytic section moments and segmented 16-point Gaussian quadrature; calling it a fully closed-form contact solver would be inaccurate. The interior-coverage proof can also establish the same canonical midpoint node before polygon clipping; cells without that proof retain the clipped rule.

The plane integral returns zero overlap and zero derivatives when the plane is strictly separated from the complete carcass support, with a numerical margin. The early return requires representable inputs and derived quantities; near-contact cases and unrepresentable arithmetic retain the generic evaluation. For a vertical column, convexity permits any point proved strictly interior to serve as the interior endpoint of both contour-root brackets. The carcass geometry, boundary solver and tolerances remain unchanged.

SIMD column evaluation preserves all sixteen Gauss samples and their weights. Axial coefficient α and radial coefficient β express the same gradient; its angular integral is reconstructed from the spatial-gradient sum and two weighted first vertical moments, one for each horizontal component. Endpoint-motion contributions are applied after reducing the samples through the same algebraic factorization of the integral. SSE2 processes two samples in parallel and AVX2 processes four. Regrouping changes floating-point operation order and can introduce rounding differences. The AVX2 path is selected only after checking CPUID, OSXSAVE and enabled XMM/YMM state in XCR0; configurations without those capabilities retain the compatible SSE2 or scalar path.

Each preparation cache retains up to four exact recent plans containing fixed geometry, interval unions, and quadrature nodes; the fine-query and enlarged-window routes use separate caches. Reuse requires matching geometry order, support identities, immutable-topology identity, lower planes, foundation height, spacing, and cell bounds. Material and rigid-velocity data remain current. Tire overlap and its derivatives are recomputed from the current physical state; the cache does not retain an old contact force.

Nonplanar or mixed preparation plans may retain a fixed world-XY support envelope for the rounded carcass. Two directions remain fixed for that plan; their four support limits include a 0.04 m margin. A cache hit additionally requires the complete current carcass to fit inside those limits. An entire cell can be omitted before clipping only when one retained support plane separates it from every admitted carcass pose. Group membership, interval ownership and quadrature in contributing cells remain unchanged. The single planar immutable Body0 case keeps its original domain for direct plane integration and its consistency correction.

After evaluating the stationary foundation, a zero foundation overlap and a strict lower support bound above every queried top vertex can prove the complete tire disjoint from all selected solids. That proof permits an inactive return before preparation. It follows the complete solid query; it does not make the existence of a tall obstacle depend on a wheel-height cap.

An additional path may prepare immutable terrain in windows aligned to five-cell blocks when no scene solids are present. Eligibility requires unique identities in one valid immutable topology, manifold groups, and real geometric-neighbor evidence for every canceled edge. A connected pair with a sufficiently strong non-coplanarity witness must remain present in the current fine query. This prevents window expansion from introducing a direct-plane or plane-consistency case that the fine query would not select.

Edge cancellation is checked against actual shared vertices, not only rounded coordinate keys. A retained window evaluates only nodes that can touch the current carcass, while its geometry, boundaries and eligibility certificates remain valid. It does not apply the narrower XY envelope used by an ordinary

preparation plan. Missing proof or any scene solid selects the fine-query path. That path may reuse eligible per-cell preparation as described below; selecting it does not require rebuilding every cell. Neither route reuses contact forces from a previous pose.

A separate per-wheel cell-preparation memo stores world-XY quadrature nodes, geometric interval endpoints, interval ownership and foundation-cut flags. It is distinct from the recent complete plans and triangle-selection caches. Eligible immutable-terrain windows and eligible mixed fine queries share this memo between their fine and enlarged-window caches. A mixed fine query can retain cells belonging to immutable terrain and stationary supported scene solids. Cells containing moving scene geometry keep direct preparation; scene coverage still uses the clipped seven-point triangle rule rather than adopting the terrain-interior midpoint shortcut.

Reuse requires an exact key for the relevant geometry, current candidate ordering and grouping, support-body identity, immutable owner, upper and lower boundaries, foundation, grid location and spacing, and boundary-halo coverage. Mixed fine keys additionally distinguish the supported floating-point control settings. Hash matches are verified against the complete stored key. Stored source ordinals are remapped to the current candidate list; current materials, references and rigid velocities are not frozen in the memo. Tire position, orientation, overlap, derivatives, elastic energy and contact wrench are evaluated anew at each call. This per-cell control key does not add an arbitrary floating-point-context guarantee to the separate complete-plan cache.

Boundary exclusions use a bounded grid of cells intersecting the boundary halo. The lookup preserves the same inclusive edge-bounds predicate and numerical margins as the direct boundary scan. When that grid cannot be prepared within its limits, the direct scan remains available. Thus accelerating the proof does not widen the region admitted to midpoint integration or remove a real support edge.

The cell memo has a finite per-wheel storage budget shared by its eligible routes. Each record is limited to 64 KiB. Under storage pressure, retirement occurs before cell views are obtained, with at most eight records and 64 KiB retired per contact lookup; storing a record does not evict a live view. If allocation, admission or reuse is unavailable, the cell is prepared directly. These limits bound memo storage and its retirement work, not the total cost or memory of every contact query. The same spatial nodes, weights, interval union and physical evaluation remain in use on either route.

When the selected immutable geometry is a single connected planar group with an admissible lower boundary, a plane-consistency correction is blended into its generic integral:

$$J = J_Q + B(f)\,[J_P - J_{Q,P}],$$

$$t = \mathrm{clip}[(f - 0.1)/0.8, 0, 1], \qquad B(f) = t^3(10 - 15t + 6t^2).$$

The fraction f comes from smooth coverage weights based on an inscribed ellipsoid. J_P is the plane/slab evaluation and $J_{Q,P}$ its corresponding canonical midpoint-lattice evaluation. Derivatives include the derivatives of the coverage fraction and blend function. This correction is restricted to immutable geometry; scene bodies retain the clipped polygon rule. Omitting the coverage derivatives would produce a force that is not the derivative of the implemented elastic energy.

The lateral bands retained in contact data summarize materials and representative points. They are metadata, not independent normal springs. The older height-sampler kernel remains available to laboratory callers and as a reported compatibility fallback for unsupported scene shape types; it is a distinct contact interface from the exhibition solid path.

7.3 Equivalent penetration and elastic energy

The overlap is normalized by $R_t^2 w/2$. A monotone, calibrated plane-depth relation is inverted to obtain raw equivalent penetration:

$$\delta_{\mathrm{raw}} = R_t\, D\!\left(\frac{J}{R_t^2 w/2}\right).$$

The inverse uses a precomputed table with 4,096 intervals, monotone cubic interpolation, and safeguarded Newton/bisection refinement. It makes the flat-plane force law correspond to the intended tire stiffness while retaining the geometric volume response on general support.

Let $\delta_{\max} = 0.3 R_t$. The elastic energy factor is

$$E_\delta = \begin{cases} \delta_{\mathrm{raw}}^2, & \delta_{\mathrm{raw}} \le \delta_{\max}, \\ 2\delta_{\max}\delta_{\mathrm{raw}} - \delta_{\max}^2, & \delta_{\mathrm{raw}} > \delta_{\max}. \end{cases}$$

For a rigid substrate,

$$U_t = \tfrac{1}{2} k_t E_\delta, \qquad \delta_c = \min(\delta_{\mathrm{raw}}, \delta_{\max}).$$

The continuation above the cap is linear in raw penetration. It limits the elastic-force coefficient without replacing the energy gradient with a discontinuous flat energy.

7.4 Normal wrench and moving supports

Let $\mathbf{G} = \partial\delta_{\mathrm{raw}}/\partial\mathbf{x}_w$ and $\mathbf{G}_\theta$ be the angular derivative for a small wheel rotation. On a flat floor, $\mathbf{G}$ points downward. Define

$$\mathbf{n} = -\mathbf{G}/\|\mathbf{G}\|, \qquad \dot\delta = \mathbf{G}\cdot\mathbf{v}_w + \mathbf{G}_\theta\cdot\boldsymbol{\omega}_w + \sum_s (\mathbf{G}_s\cdot\mathbf{v}_s + \mathbf{G}_{\theta,s}\cdot\boldsymbol{\Omega}_s).$$

The derivatives for each support describe its contribution to relative contact motion. The normal force magnitude is

$$N = \max[0, (k_t\delta_c + c_t\dot\delta)\|\mathbf{G}\|].$$

The tensile side is clipped to zero. At no overlap, or a degenerate gradient, the contact is inactive. A load above 1 N marks a tire as grounded for its local state; other controllers deliberately use higher support thresholds.

The contact normal need not equal one triangle normal. It is the direction of the assembled contact-energy gradient. The normal force is applied with an additional pure couple:

$$\boldsymbol{\tau}_{\mathrm{extra}} = N\left[-\frac{\mathbf{G}_\theta}{\|\mathbf{G}\|} - (\mathbf{x}_p - \mathbf{x}_w)\times\mathbf{n}\right].$$

Together, the representative point force and this couple recover the normal angular work implied by the integrated energy. Treating the representative point as the only moment arm would discard this contribution.

Support force/moment derivatives are grouped by support body. When a moving prism extends below the stationary foundation, its integrated solid interval is clipped at the foundation. That artificial lower cut belongs to the foundation, rather than the moving prism. The implementation transfers the cut's signed force and moment derivatives between those supports without changing tire overlap, elastic energy, or the total contact wrench. This matters for both moving-support work and damping when the lower face intersects the floor.

The assembled reactions retain the opposite force and moment implied by the integrated potential, rather than applying only one opposite force at an averaged point. Finite quadrature, support topology changes, and partitioned rigid-body stepping still prevent a claim of exact global energy conservation.

7.5 Contact limitations

The solid path represents geometry as projected triangular patches with upper and lower affine planes. It can represent vertical side boundaries through the boundary of a patch, finite decks, and multiple separated vertical intervals. Scene extraction supports oriented boxes, convex polyhedra, and closed triangle meshes using their collision vertices and faces. A bounding box only selects candidates; it does not replace their shape. The legacy height-sampler fallback remains for unsupported implicit shape types and retains its upward-hit and height-cap limitations. Runtime diagnostics report regions that select that fallback. Native rigid collision shapes continue to protect the chassis independently of the tire contact kernel.

The 0.04 m cells, finite quadrature, geometric clipping and plane-group tolerances, inverse-table resolution, contact activation thresholds, and scene-shape coverage are part of the numerical model. Small features, inconsistent mesh winding, open meshes, and support-identity transitions can expose limitations. A seven-point rule is exact for polynomials through degree five; the actual tire integrand is not generally such a polynomial. Smooth contact-potential construction does not establish convergence for every possible obstacle or globally smooth hybrid dynamics.

No finite-element strain field, belt modes, carcass hysteresis spectrum, tread-block vibration, local contact pressure map, or tread material failure is solved.

8. Tire traction, wheel rotation, heat, and wear

8.1 Pressure, vertical stiffness, and footprint estimate

Tire gas uses a fixed-volume ideal-gas scaling. For cold gauge pressure p_0 specified at ambient temperature T_a, current tire temperature T_t, and pressures in bar,

$$p_t = (p_0 + 1.01325)\frac{T_t + 273.15}{T_a + 273.15} - 1.01325.$$

Define $p_f = \mathrm{clip}(p_t/2.2, 0.3, 2)$. Vertical stiffness and nominal slip stiffnesses become

$$k_t = k_{t0}(0.35 + 0.65p_f), \quad K_x = K_{x0}p_f^{0.15}, \quad K_y = K_{y0}p_f^{0.35}.$$

Defaults are $k_{t0} = 200{,}000$ N/m, $c_t = 350$ N s/m, $K_{x0} = 90{,}000$ N, and $K_{y0} = 78{,}000$ N. The loaded rolling radius and estimated footprint area are

$$R_l = R_t - \min(\delta_c, 0.3R_t), \qquad A_{\mathrm{est}} = \frac{N}{\max(10^5 p_t, 10^4)}.$$

This area estimate is a load/pressure diagnostic. It does not set the spatial contact distribution or replace the weighted-volume normal model. Temperature is one lumped carcass/tread state; gas volume changes, tire breathing, gas leakage, and heat exchange between separate gas and rubber states are absent.

8.2 Contact velocities and slip

The wheel-center velocity includes chassis translation and rotation, axle Jacobian velocity, and axle rotation about its center. For moving support, the local support rigid velocity field is subtracted consistently at contact. The tire's forward direction is the steered direction projected into the contact tangent plane; lateral direction is $\mathbf{n} \times \mathbf{e}_x$.

With tangent velocities v_x, v_y, spin ω, and loaded radius R_l, the slip variables are

$$\kappa = \frac{\omega R_l - v_x}{\max(|v_x|, |\omega R_l|, 0.75)}, \qquad \alpha = \operatorname{atan2}(v_y, \max(|v_x|, 0.5)).$$

The 0.75 m/s and 0.5 m/s denominators regularize standstill and reversing motion. They affect creep and low-speed turning and should not be mistaken for measured tire properties.

The load-, temperature-, wear-, and geometry-dependent friction coefficient is

$$\mu = \mu_s \left(\frac{\max(N, 1)}{5000} \right)^{-0.08} \exp\left[- \left(\frac{T_t - 40}{140} \right)^2 \right] (1 - 0.25 w_t)\, \eta_{\text{tread}}.$$

Here $w_t \in [0, 1]$ is wear and

$$\eta_{\text{tread}} = \sqrt{\max[0, 1 - (\mathbf{n} \cdot \mathbf{a})^2]}.$$

The last factor suppresses driven tread traction for predominantly axial side contact. The temperature multiplier is a broad empirical curve centered at 40°C. The load exponent is an empirical load-sensitivity law.

8.3 Combined-slip brush saturation and relaxation

The unsaturated tangent demand is

$$\mathbf{q}_t = (K_x \kappa, -K_y \tan \alpha), \qquad Q = \|\mathbf{q}_t\|, \qquad L_\mu = \max(0, \mu N).$$

For $L_\mu > 0$,

$$s = \operatorname{clip}\left(\frac{Q}{3 L_\mu}, 0, 1 \right), \qquad \mathbf{F}^* = L_\mu [1 - (1 - s)^3] \frac{\mathbf{q}_t}{Q}.$$

Zero demand gives zero force. This cubic brush-style saturation transitions from approximately linear stiffness to the combined friction limit. It is an isotropic combined-force budget; there are no separately fitted longitudinal/lateral peak-slip curves or measured aligning-moment coefficients.

The persistent tangent force follows an exponential spatial relaxation:

$$\mathbf{F}^+ = \mathbf{F} + \lambda(\mathbf{F}^* - \mathbf{F}), \qquad \lambda = 1 - \exp[-\max(|v_x|, 2) h_m / \ell_r],$$

with default $\ell_r = 0.22$ m. The 2 m/s floor prevents indefinitely frozen tire force at rest. After the spin solve, the final force is projected into $\|\mathbf{F}\| \le \mu N$; changing load cannot leave the stored relaxation force outside the current friction circle.

8.4 Implicit wheel spin and rolling resistance

Each wheel has default spin inertia $I_w = 2.7\ \mathrm{kg\,m^2}$. The new angular velocity solves

$$I_w \frac{\omega^+ - \omega}{h_m} = T_d - R_l F_x(\omega^+) - (T_b + T_{\mathrm{rr}}) \tanh(\omega^+/0.12).$$

The dependence of κ, brush force, and relaxed force on ω^+ is included in this scalar solve. A bounded bracket and 32 bisection iterations avoid a fully explicit stiff wheel/tire update.

Rolling resistance is

$$T_{\mathrm{rr}} = N\,(c_{\mathrm{rr}} + 0.25 z_s/R_t)\,R_l p_f^{-0.35},$$

where $z_s = 0$ on the active rigid course and c_{rr} is the surface rolling coefficient. The hyperbolic tangent provides a smooth sign near standstill. Consequently, brake/rolling torque approaches zero at exactly zero spin; static holding emerges through the coupled implicit tire/brake solution rather than an ideal algebraic wheel lock.

The axle receives the contact force $F_x\mathbf{e}_x + F_y\mathbf{e}_y + N\mathbf{n}$ and the contact couple. The chassis also receives the modeled reaction associated with wheel spin acceleration, $-I_w(\omega^+ - \omega)\mathbf{e}_y/h_m$. This is a reduced rotor reaction. A complete gyroscopic multibody model of steering knuckles, rotating shafts, and flexible tires is not present.

8.5 Tire temperature and wear

A slip/rolling dissipation proxy is

$$H_t = |F_x(\omega R_l - v_x)| + |F_y v_y| + T_{\mathrm{rr}}|\omega|.$$

The thermal and wear updates use

$$\dot{T}_t = \frac{0.65 H_t - 45(T_t - T_a)}{18000}, \qquad \dot{w}_t = H_t/(2\times 10^9).$$

The thermal capacity is 18,000 J/K and the constant cooling coefficient is 45 W/K. Temperature is clamped between ambient and 180°C; wear is clamped to $[0, 1]$. These empirical laws do not resolve tread thickness, temperature gradients, convective flow, abrasion particles, chemical aging, tire failure, or changes to geometric radius from tread loss.

9. Engine, clutch, transmission, and differentials

9.1 Engine torque and loss map

The engine is one rotor with default inertia $I_e = 0.32\ \mathrm{kg\,m^2}$. Its indicated drive torque is a piecewise-linear normalized RPM map:

Reference RPM	Torque factor
0	0.40
850	0.52
1,500	0.78

Reference RPM	Torque factor
2,400	0.96
3,200	1.00
4,200	0.94
5,000	0.84
5,800	0.69

The lookup uses $5800\,n/n_{\max}$, so changing maximum RPM stretches the reference speed axis. Produced torque is

$$T_e = T_{\text{peak}} f(n)\, u_{\text{eff}}.$$

The default peak is 360 N m. The effective throttle includes an idle controller:

$$u_{\text{eff}} = \max\{u, \text{clip}[0.0025(n_{\text{idle}} - n), 0, 0.38]\}.$$

Produced torque is cut above maximum RPM or when the engine is stopped. Default idle and maximum speeds are 850 and 5,800 RPM. Loss torque, for running angular speed ω_e, is

$$T_{\text{loss}} = 12 + 0.08\omega_e + (1-u)0.025\omega_e.$$

These coefficients define an aggregate speed-dependent friction/pumping closure, not cylinder pressure, throttle airflow, turbocharger dynamics, or combustion thermodynamics. Reported shaft power uses $P_{\text{kW}} = T\,n\pi/30000$.

9.2 Ratios and shifting

Forward ratios default to $4.16, 2.37, 1.56, 1.15, 0.86, 0.67$, reverse magnitude to 3.8, final drive to 3.73, and low range to 2.48. Let signed overall ratio be

$$G = g_{\text{selected}}\, g_{\text{final}}\, g_{\text{range}}.$$

Neutral and free-coast operation disconnect the drive path. Let $\bar{\omega}_F = (\omega_{FL} + \omega_{FR})/2$ and $\bar{\omega}_R = (\omega_{RL} + \omega_{RR})/2$. The transmission-side wheel speed is

$$\omega_s = G \begin{cases} \bar{\omega}_R, & \text{4x2, Rear selected}, \\ \bar{\omega}_F, & \text{4x2, Front selected}, \\ (\bar{\omega}_F + \bar{\omega}_R)/2, & \text{4WD}. \end{cases}$$

Rear is the default 4x2 axle. The selected axle determines both positive drive and back-driving through the clutch or converter. The 4x2 preference has no effect on the 4WD shaft-speed equation. No explicit propeller-shaft torsional state is integrated.

Manual sequential shifts disconnect torque for approximately 0.22 s. Automatic shifts use throttle-dependent up/down thresholds,

$$n_{\text{up}} = 2500 + 2600u, \qquad n_{\text{down}} = 1100 + 650u,$$

with approximately 0.30 s disconnection. Direction-change and hardware configuration interlocks prevent some changes while moving or loaded. They are state-transition guards, not synchronizer or gear-tooth collision models.

9.3 Clutch and engine update

For manual or locked-converter coupling, define

$$D_{\text{eff}} = \frac{1}{I_e} + \frac{G^2}{N_d I_w}.$$

Here N_d is the number of driven wheels. The implicit clutch torque request is

$$T_c^* = \frac{\omega_e - \omega_s + h_m(T_e - T_{\text{loss}})/I_e}{h_m D_{\text{eff}}}, \qquad T_c = \text{clip}(T_c^*, -T_{\max} c, T_{\max} c).$$

The clutch engagement $c \in [0, 1]$ comes from the driver or clutch assistance; default capacity is 650 N m. The reflected wheel inertia regularizes the coupling without solving a separate elastic shaft. Clutch assistance uses $c = \text{clip}[(n - 700)/600, 0, 1]$, with shift and free-coast overrides.

The rotor update is

$$\omega_e^+ = \max[0, \omega_e + h_m(T_e - T_{\text{loss}} - T_c)/I_e].$$

In manual operation without clutch assistance, falling below 450 RPM stalls the engine. A stopped engine spin state decays at 20 rad/s². There is no clutch temperature, plate wear, flywheel torsion, starter battery model, or fuel consumption.

9.4 Automatic torque converter

For the unlocked automatic converter, the speed ratio is

$$\nu = \text{clip}[\omega_s / \max(\omega_e, 1), 0, 1.2].$$

The pump torque uses the signed slip closure

$$T_p = \text{clip}[0.012 \omega_e^2 (1 - \nu)|1 - \nu|, -T_{\max}, T_{\max}].$$

The turbine torque is $T_t = T_p \, m(\nu)$, with

$$m(\nu) = \begin{cases} 2.05 - 1.05 \, \text{clip}(\nu / 0.85, 0, 1), & \nu < 1, \\ 1, & \nu \geq 1. \end{cases}$$

Lockup is allowed in third gear and above, with $\nu > 0.87$, throttle below 0.75, and no active shift. The engine reacts to pump torque; the wheels receive turbine torque. This is an empirical converter map with no fluid state, stator one-way-clutch dynamics, oil temperature, or converter cooling model.

9.5 Efficiency, axle differentials, and 4WD transfer

The wheel-side torque is $T_t G \eta$ during forward power transfer and $T_t G / \eta$ during back-driving, with default $\eta = 0.92$. This signed convention produces loss in either power direction.

For an axle input torque T_a, open differential output is equal torque $T_a/2$ at each wheel. Speed difference produces a transfer torque

$$C_d = \mathrm{clip}[K_d(\omega_R - \omega_L), -C_{\max}, C_{\max}],$$

$$T_L = T_a/2 + C_d, \qquad T_R = T_a/2 - C_d.$$

An open differential has zero transfer capacity. The limited-slip setting uses $K_d = 25\ \mathrm{N\,m\,s}$ and $C_{\max} = 0.30|T_a| + 30$ N m. The locked setting uses $K_d = 600\ \mathrm{N\,m\,s}$ and a 2,500 N m capacity. Both conserve the axle's total allocated torque while resisting speed difference. A lock is a finite-capacity dissipative coupling, not a holonomic equality of left and right speeds.

In 4WD, a front/rear coupling adds

$$C_4 = \mathrm{clip}[400(\bar{\omega}_R - \bar{\omega}_F), -1800, 1800],$$

$$T_F = T_{\mathrm{drive}}/2 + C_4, \qquad T_R = T_{\mathrm{drive}}/2 - C_4.$$

Equal front/rear means give $C_4 = 0$ and a 50/50 base split. The correction has equal and opposite axle contributions with magnitude at most 1,800 N m; it is a speed-difference reaction rather than a direct estimate of available tire grip. The 4WD relation models a finite transfer coupling between axle-speed means; it does not model a permanently driven center differential or a detailed transfer-case gear train.

In 4x2, $(T_F, T_R) = (0, T_{\mathrm{drive}})$ for Rear and $(T_{\mathrm{drive}}, 0)$ for Front. The non-driven axle receives zero net input torque from the engine. Left/right differential reactions can still resist wheel-speed differences on either axle. The retained 4x2 axle preference has no effect on the 4WD equations.

Changing the active 4x2 axle requires total body speed at most 0.15 m/s, angular speed at most 0.15 rad/s, throttle at most 0.05, and $|\bar{\omega}_F - \bar{\omega}_R| \le 2$ rad/s. The transmission must be in neutral, have clutch engagement below 0.15, or be automatic with service-brake input above 0.2. Rejected changes leave parameters and integrated state intact; accepted mode changes preserve gear, engine state, wheel motion, temperature, wear, and elapsed time. Selecting the axle changes the drivetrain coupling and torque path, while the steering geometry and rear parking-brake location retain their original definitions.

The engine angular-acceleration reaction is applied about the body forward axis. This is the chosen reduced engine-reaction axis, not a separately modeled physical crankshaft installation.

10. Braking and longitudinal assistance

10.1 Hydraulic pressure and distribution

The brake request combines the pedal with active assistance. Normalized hydraulic pressure p_b follows

$$p_b^+ = p_b + (1 - e^{-h_m/\tau_b})(p_{\mathrm{req}} - p_b),$$

with default apply/release time constants 0.07/0.05 s. This is one aggregate pressure state, not individual hydraulic circuits, wheel-cylinder displacement, or a fluid-compressibility model.

Electronic brake distribution uses measured front and total normal load:

$$f_F^* = \mathrm{clip}[f_{F0} + N_F/N_{\mathrm{total}} - 0.5, 0.5, 0.85],$$

when total load exceeds 100 N; the target is filtered with a 0.08 s time constant. The default base front fraction is 0.64. Without EBD, the base split is used. Service torque per wheel is

$$T_{b,i} = T_{\text{nominal}}\, p_b\, a_i\, f_{T,i} \begin{cases} 2f_F, & i \text{ front}, \\ 2(1-f_F), & i \text{ rear}. \end{cases}$$

The default nominal torque is 2,600 N m. The factors of two make this parameter a reference per-wheel torque at a 50:50 axle split. Rear parking brake adds up to 2,200 N m per rear wheel independently of the service-pressure and ABS demand.

10.2 ABS

For $|v_x| \geq 2$ m/s and nontrivial pressure, the ABS monitor uses

$$\rho_{\text{ABS}} = |\omega R_l| / \max(|v_x|, 0.1).$$

Below 0.80, the wheel's pressure multiplier falls at 20 s⁻¹, with a floor of 0.18. Above 0.90 it recovers at 8 s⁻¹, to a ceiling of one. Intermediate values hold the multiplier. Below the speed/pressure thresholds the multiplier resets. ABS is reported active below 0.98.

This is threshold modulation of an effective brake torque, not a resolved valve/pump hydraulic cycle or an estimator of vehicle speed from wheel sensors. Parking torque is outside this ABS service-torque modulation.

10.3 Brake heating and fade

For total applied brake torque, including parking torque, dissipation is

$$H_b = T_b |\omega \tanh(\omega / 0.12)|,$$

$$\dot{T}_b = \frac{H_b - (35 + 4|v_x|)(T_b - T_a)}{18000}.$$

The default heat capacity is 18,000 J/K. Temperature is clamped between ambient and 1,200°C. The service-brake fade factor is

$$f_T = 1 - 0.65\ \text{clip}\left(\frac{T_b - T_{\text{start}}}{T_{\text{end}} - T_{\text{start}}}, 0, 1\right),$$

with default fade onset/end at 350/700°C. The fully faded service factor is therefore 0.35. This single-state model omits disc/pad temperature separation, fluid boiling, thermal expansion, rotor cracking, and material-specific friction recovery.

10.4 Grade estimation and hill-start hold

Hill assistance requires at least two tire contacts above 100 N, a sufficiently upright body, and an upward weighted support normal. Grade is the angle of the body's forward tangent direction projected onto the support plane, filtered with a 0.15 s time constant.

Hill-start hold can capture brake pressure above 0.25 when grade exceeds 3° and speed is below 0.18 m/s. The default hold duration is 2 s. It releases on timeout, loss of valid support, free coast, sufficient motion, or an uphill launch request whose drive-force estimate exceeds $1.1 M_{\text{total}} g |\sin\theta|$. Normal timed release reduces stored hold pressure at $1/0.22$ per second; loss of support/free coast clears it directly.

This controller retains a pressure demand; it does not constrain the chassis to remain fixed on the grade.

10.5 Hill-descent control

Hill-descent control operates in low range, in a nonneutral gear, with valid support, downhill travel relative to the selected direction, and no significant throttle, service-brake, or parking-brake override. Entry is limited to low speed. Its default target is 2 m/s.

For speed error $e = |v| - v_{\text{target}}$, the demand combines grade feed-forward and PI-like feedback:

$$u_{\text{ff}} = \frac{M_{\text{total}} g |\sin\theta| R_t}{4 T_{\text{nominal}}}, \qquad \dot{I} = 0.12 e,$$

$$u_{\text{HDC}} = \text{clip}(u_{\text{ff}} + 0.18 e + I, 0, 1).$$

The integral is bounded to $[-0.5, 0.75]$ and reset below 55% of target speed; demand is also withheld below that threshold. It passes through normal pressure response, EBD, ABS, and thermal fade. The controller cannot create extra tire adhesion or guarantee target speed on every grade.

11. Steering, aerodynamic drag, and residual damping

11.1 Steering actuator and Ackermann geometry

Controller steering uses a dead zone and a signed power curve $u_s = \text{sgn}(u)|u|^{\gamma}$, with default exponent 1.35. Keyboard demand uses its normal command values. Except for a calibrated direct wheel input, steering then follows a critically damped actuator:

$$\ddot{\delta} + 2\omega_n \dot{\delta} + \omega_n^2(\delta - \delta_{\text{target}}) = 0, \qquad \omega_n = 2/\tau_s.$$

For constant target during a substep, the analytic update is formed from

$$e = \delta - \delta_{\text{target}}, \quad b_s = \dot{\delta} + \omega_n e,$$

$$\delta^* = \delta_{\text{target}} + (e + b_s h_m) e^{-\omega_n h_m}, \qquad \dot{\delta}^* = (\dot{\delta} - \omega_n b_s h_m) e^{-\omega_n h_m}.$$

The resulting position increment and rate are limited by the maximum steering rate and travel. Defaults are $\tau_s = 0.10$ s, maximum road angle 34°, and maximum rate 1.8 rad/s. Direct wheel input bypasses this actuator smoothing and requests road angle directly after calibration.

For effective central angle δ, the turn radius is $R_{\text{turn}} = L/\tan\delta$. Front wheel angle at lateral side sign s_i is

$$\delta_i = \text{atan}\left(\frac{L}{R_{\text{turn}} - s_i t/2}\right).$$

Rear wheels do not steer. The visible steering wheel follows the physical road-angle state at a 13.2:1 animation ratio. This does not add a rack, steering-column inertia, caster trail, scrub-radius force, or kingpin compliance to the mechanical plant.

11.2 Aerodynamic drag

The chassis experiences a force at its center of mass:

$$\mathbf{F}_D = -\tfrac{1}{2}\rho_{\text{air}} C_D A \|\mathbf{V}\| \mathbf{V},$$

with fixed $\rho_{\text{air}} = 1.225\ \text{kg}\,\text{m}^{-3}$ and default combined drag area $C_D A = 0.95\ \text{m}^2$. The velocity is relative to a stationary ambient frame. Weather wind settings drive visual dust/exhaust motion; they do not change this aerodynamic relative velocity.

There is no aerodynamic lift, downforce, side-force coefficient, yawing moment, underbody ground effect, altitude-dependent density, or speed-dependent body aero map.

11.3 Residual angular loss

The vehicle kernel adds $\boldsymbol{\tau}_d = -18\boldsymbol{\Omega}$ in SI units as a residual structural/bearing angular damping term. The runtime body configuration also assigns Chaos angular damping of 0.12. These are separate dissipative mechanisms. Native linear damping is zero; longitudinal losses arise from the modeled aerodynamic, rolling, drivetrain, brake, and contact mechanisms.

12. Reaction control system

12.1 Nozzles and physical wrench

There are four vertical and four lateral nozzles, arranged in front/rear and left/right pairs. All are fixed in the chassis frame. Vertical thrust directions are body-up. Each lateral nozzle's reaction force points inward from its side of the body.

At default vehicle dimensions, vertical locations are approximately $x = \pm 2.04$ m, $y = \pm 0.58$ m, with z about 0.063 m at the front and 0.073 m at the rear. Lateral locations have the same longitudinal offsets, lateral magnitude about 0.814 m, and height 0.15 m. A shared body-fitting function scales longitudinal/lateral placement with wheelbase and track and adjusts local body clearance with enlarged tires. Physical force positions therefore follow the fitted visible nozzle openings.

For actual scalar forces F_i, local unit directions $\mathbf{d}_i$, and locations $\mathbf{r}_i$,

$$\mathbf{F}_{\text{RCS}} = \mathbf{R}\sum_i F_i \mathbf{d}_i, \qquad \boldsymbol{\tau}_{\text{RCS}} = \mathbf{R}\sum_i (\mathbf{r}_i - \mathbf{c}) \times F_i \mathbf{d}_i.$$

No independent artificial roll or pitch torque is added by the flight controller. The available translational and angular accelerations depend on actual lever arms, mass, full body inertia, pressure, flow saturation, valve history, and other physical loads.

12.2 Ideal storage and residual gas

Let C be usable gas capacity, $m \in [0, C]$ usable gas remaining, and P_f, P_c, P_r full, cutoff, and regulator pressures. The model uses

$$m_{\text{residual}} = \frac{C P_c}{P_f - P_c}, \qquad P = P_c + (P_f - P_c)\frac{m}{C}.$$

The installed gas mass is $m + m_{\text{residual}}$. Reaching zero usable gas leaves residual gas at cutoff pressure. The default dry equipment mass is 80 kg and usable capacity is 100 kg. At default 300/20 bar full/cutoff settings, residual gas is approximately 7.14 kg.

This linear pressure relation is consistent with fixed-volume isothermal ideal storage. For air constant $R_a = 287.05\ \mathrm{J\,kg^{-1}\,K^{-1}}$, ambient absolute temperature T, and pressure in pascals, an implied volume is

$$V_{\text{tank}} = (C + m_{\text{residual}}) R_a T / P_f.$$

Tank geometry is not solved or used to derive the equipment mass. There is no blowdown cooling, wall thermal capacitance, real-gas compressibility correction, pipe pressure loss, regulator spool dynamics, or detailed choked-flow/nozzle expansion. Ideal-gas behavior at these storage pressures is a deliberate simplification.

12.3 Regulation, valves, shared flow, and consumption

The pressure authority multiplier is

$$\eta_P = \operatorname{clip}\left(\frac{P - P_c}{P_r - P_c}, 0, 1\right).$$

Above regulator pressure, nominal force authority is available; below it, authority falls linearly to zero at cutoff. Default nominal force is 10,000 N per vertical nozzle and 2,000 N per lateral nozzle.

For requested aperture $u_i > 0$, valve state a_i follows a first-order response. Over pneumatic substep h_r,

$$a_i^+ = u_i + (a_i - u_i) e^{-h_r/\tau_v},$$

$$\bar{a}_i = u_i + (a_i - u_i) \frac{\tau_v}{h_r} (1 - e^{-h_r/\tau_v}).$$

Force uses the interval-average opening $\bar{a}_i$. A zero target closes the valve immediately, rather than following an exponential tail. Default opening time constant is 0.025 s.

Before the shared limit, $F_i^* = \bar{a}_i F_{i0} \eta_P$. Total force availability is

$$F_{\text{available}} = \min(\dot{m}_{\max} v_e, m v_e / h_r),$$

where v_e is equivalent exhaust velocity. All requests are scaled by one factor,

$$\sigma = \min\left(1, \frac{F_{\text{available}}}{\sum_i F_i^*}\right), \qquad F_i = \sigma F_i^*,$$

with zero force for zero request. Default $\dot{m}_{\max} = 100$ kg/s and $v_e = 500$ m/s. The consumption and accumulated scalar impulse are

$$\Delta m = \frac{h_r}{v_e} \sum_i F_i, \qquad \Delta \mathcal{J} = h_r \sum_i F_i.$$

This is an effective $F = \dot{m} v_e$ closure. Exhaust velocity subsumes the omitted nozzle momentum/pressure details. Simultaneous lateral demand can reduce vertical force through the shared flow limit. The recorded scalar impulse is the sum of nozzle impulse magnitudes, not the magnitude of net vehicle impulse.

12.4 Manual allocation, trim, and damping

Let collective be c, roll request r, pitch request p, and nozzle side signs $s_x = \pm 1, s_y = \pm 1$. Without inertia compensation, the vertical allocation starts from multiplicative weights of the form

$$u_i = c \max\left[0, 1 + (1-d)\left(s_x c_x / a + s_y c_y / b\right)\right](1 - s_y r)(1 - s_x p),$$

where a, b are representative longitudinal/lateral nozzle offsets and $d = \max(|r|, |p|)$. Pitch authority scales and clamps the pitch request before this legacy allocation. CG compensation fades toward extreme attitude commands. A full command can close an opposite bank exactly.

The openings are normalized to at most one and their sum is bounded by $4c$. Differential control redistributes the collective aperture budget rather than adding an unrestricted extra lift request.

With centered attitude input and positive collective, CG trim distributes vertical force about the current mass center. Optional angular damping requests approximately

$$\boldsymbol{\tau}_{\text{trim}} = \mathbf{I}_C(-k_\omega \omega_x, -k_\omega \omega_y, 0)^{\mathsf{T}} + \boldsymbol{\omega} \times (\mathbf{I}_C \boldsymbol{\omega}) - \boldsymbol{\tau}_{\text{lateral,nominal}}.$$

The default $k_\omega = 6\ \text{s}^{-1}$. This moment request is converted to vertical aperture differences using nominal thrust and lever arms, then constrained by the same aperture budget. Actual pressure, valve lag, and common flow saturation can reduce the achieved moment.

This damping acts on angular rates when the attitude stick is centered. It does not impose a level attitude, yaw rate, angular velocity target during a manual deflection, or a pose correction. With collective zero, vertical attitude control has no authority.

12.5 Pitch inertia compensation

The optional compensated allocator uses the full sprung tensor to form each unit-force angular response:

$$\mathbf{b}_i = \mathbf{I}_C^{-1}[(\mathbf{r}_i - \mathbf{c}) \times \mathbf{d}_i].$$

Its roll and pitch rows map the four vertical apertures to a nominal angular-acceleration contribution. It first constructs the roll-only baseline. A symmetric positive/negative roll allocation at the magnitude of the pitch input establishes a reference angular authority; signed pitch request and the configurable authority factor scale that reference.

The four-variable allocation enforces aperture bounds, the total budget $\sum_i u_i \le 4c$, the baseline roll response, and exact zero openings required by a full roll command. It finds the feasible pitch-response interval, clamps the request to that interval, and maximizes remaining collective lift at the chosen roll/pitch response. A tie selection favors the baseline.

The implementation enumerates vertices of this small bounded problem rather than running a general optimizer. Reported pitch reference, request, allocated contribution, saturation, and roll-allocation error describe this feed-forward allocation. They are not measurements of the complete vehicle's angular acceleration. The body tensor used here does not add both articulated axle inertias as a single rigid tensor; support reactions and gyroscopic motion remain part of the actual response.

12.6 Reserve and hover estimate

Reserve is entered when usable gas falls to $m \le C f_{\text{reserve}}$, with default fraction 0.25. Ordinary lateral demands are inhibited there unless reserve use is explicitly allowed. Vertical thrust remains available. The automatic roll controller requests reserve access. Reserve is a control policy on the same tank, not a second

independent gas volume.

The hover display compares total weight, including unsprung mass, against available body-up force. Its required force is

$$F_{\mathrm{req}} = \frac{M_{\mathrm{total}} g}{\max(0.01, \mathbf{u} \cdot \mathbf{e}_z)}.$$

If feasible, indicated collective is $F_{\mathrm{req}}/(4F_{v0}\eta_P)$. Otherwise it indicates a value above one using the flow-limited maximum. It assumes a steady upright thrust balance and cannot predict ground clearance, transient valve response, attitude-control allocation, vertical speed arrest, or simultaneous lateral-flow consumption.

12.7 Compressor, battery, and fast refill

Recharge requires installed equipment, disarmed RCS, no current thrust, automatic recharge enabled, remaining capacity, and a valid grounded/upright condition. In the wrapper, that condition requires at least two sufficiently loaded wheels and body-up world component above 0.5.

Physical compressor mode uses isothermal compression work per captured kilogram:

$$w_c = \frac{R_a T}{\eta_c} \ln\left(\max[P/101325, 1.001]\right).$$

The available energy in a substep is

$$\Delta E_{\max} = \min(1000 P_{\mathrm{comp,kW}} h_r,\ 3.6 \times 10^6 E_{\mathrm{battery,kWh}}).$$

A first mass estimate uses current pressure, then a midpoint-pressure correction limits added mass and records compression work. Power and remaining battery energy both constrain refill. Defaults are 30 kW, efficiency 0.5, and 30 kWh auxiliary energy.

The compressor is an external powered subsystem represented by its work relation and finite energy inventory. There is no battery voltage, current, internal resistance, temperature, motor torque curve, compressor displacement map, or compressor-generated vehicle heat load.

Fast recharge is enabled by default as an experimental convenience. It replaces compression with a virtual air source:

$$\Delta m_{\mathrm{fast}} = \min(C - m, C h_r / t_{\mathrm{fast}}),$$

with default $t_{\mathrm{fast}} = 5$ s. It consumes no battery energy and does not refill the battery. Pressure and gas mass still evolve from the resulting inventory. Fast refill cannot be used to assess physical compressor efficiency, energy consumption, or tank refilling performance.

12.8 Momentum exchange and resource edits

RCS dry equipment, residual gas, and usable gas share the fixed equivalent storage volume in the mass model. Expelled gas produces the nozzle thrust described above; an extra independent rocket acceleration is not added after reducing body mass.

Capturing initially stationary ambient air must accelerate that added mass to the vehicle's storage-point velocity. With intake mass rate $\dot{m}_{\mathrm{in}}$ and storage offset $\mathbf{r}_s$,

$$\mathbf{F}_{\mathrm{intake}} = -\dot{m}_{\mathrm{in}}(\mathbf{V} + \boldsymbol{\Omega} \times \mathbf{r}_s), \qquad \boldsymbol{\tau}_{\mathrm{intake,offset}} = \mathbf{r}_s \times \mathbf{F}_{\mathrm{intake}}.$$

An additional local rotational term $-\dot{m}_{\text{in}}\mathbf{I}_{\text{box,per kg}}\boldsymbol{\omega}$ accounts for spinning up the captured mass within the equivalent box. Both physical and fast recharge retain this momentum treatment. Ambient weather wind is not an intake momentum source.

This construction handles the modeled exhaust impulse and intake assimilation while updating mass properties. It is not a complete conservation-law simulation of vehicle, tank, compressor, intake flow, and resolved exhaust together.

Changing capacity or energy settings clips existing inventories to the new limits rather than automatically filling them. A resource reset or recovery refill is an imposed initial-condition change. Such operations must be excluded from a closed energy or propellant-budget experiment.

13. Automatic roll controller

13.1 Plant, preflight estimates, and phases

Automatic roll is a hybrid feedback controller commanding the same eight nozzle apertures. It never writes chassis pose, linear velocity, angular velocity, or an artificial torque.

It estimates total mass, regulated/flow-limited lift, and roll authority

$$\alpha_{\text{roll}} \approx \frac{2|y_v - c_y|F_v}{I_{C,xx} + J_{aF} + J_{aR}}.$$

The launch-speed estimate scales from a reference mass/inertia profile and increases for inadequate thrust-to-weight margin and initial tilt. A preflight propellant budget combines launch impulse, a nominal gravity-support duration, gas needed to stay above regulator pressure, and empirical maneuver/leveling allowances. It is a conservative controller heuristic, not an optimal-control solution or a formal reachability proof.

The maneuver requires flight mode, installed RCS, estimated lift at least 1.45 times weight, estimated roll acceleration at least 6 rad/s², launch-speed target no greater than 8 m/s, and sufficient usable gas for the computed budget. Existing slope, support count, vertical velocity, or angular velocity are not themselves preflight rejection gates. The takeoff phase must physically level and accelerate the body before the main turn.

The principal sequence is:

Phase	Allowed nozzle banks	Transition basis
Takeoff	Both vertical sides	Clearance, upward speed relative to grade, upright attitude and low roll/pitch rates
Initiate	Right vertical	Roll progress reaches 22°
Assist / quarter	Right vertical and rightward lateral thrust	Progress passes 65° and 100°
Lateral only	Rightward lateral thrust	Progress reaches 163°
Inverted	All off	Progress reaches 197°
Opposite lateral	Leftward lateral thrust	Predicted braking distance reaches remaining angle
Brake / brake assist	Left vertical and leftward lateral thrust	Progress passes 303° and 327°

Phase	Allowed nozzle banks	Transition basis
Brake only	Left vertical	Progress reaches 344°
Land	Both vertical sides	Physical tire support and settling criteria

Here “leftward” and “rightward” name the vehicle reaction-force direction. A side-mounted nozzle exhausts outward while pushing inward. Controller bank labels in implementation may therefore name a different side convention.

Progress is the unwrapped change of $\operatorname{atan2}(\mathbf{r}\cdot\mathbf{e}_z, \mathbf{u}\cdot\mathbf{e}_z)$ after takeoff. The target is 360.5°. The controller cancels on a 12 s timeout or unexpected post-takeoff support before a full airborne turn. Nozzle-bank sequencing is supplemented by pitch trim, valve limits, and the normal pressure/flow model.

13.2 Takeoff and braking

Takeoff uses CG trim and proportional/derivative roll leveling to redistribute vertical banks. To enter the main roll, the vehicle must be airborne, exceed the target climb speed relative to the local grade, clear the highest sampled terrain by the launch-height margin, have world-up alignment above 0.995, and have roll/pitch rates below 0.35 rad/s.

Braking uses measured roll rate ω_x and the currently estimated braking authority:

$$\theta_{\text{stop}} = \frac{\omega_x^2}{2\alpha_{\text{brake}}} + 2\tau_v \max(0, \omega_x).$$

The second term allows for valve response. Braking begins when remaining target angle is within this estimate plus a small angular margin. Vertical braking aperture is proportional to $1.08\omega_x^2/(2\theta_{\text{remaining}}\alpha_{\text{brake}})$, clipped into $[0.03, 1]$.

This uses instantaneous estimates. Gas depletion, changes in attitude, lateral thrust moments, and saturation can alter actual stopping performance.

13.3 Terrain-relative landing

The controller samples the center and corners of the body footprint for terrain height, highest nearby support, normal, and support velocity. These are present vertical samples, not a swept collision forecast or a model-predictive terrain planner.

For a local plane normal $\mathbf{n}$, support velocity $\mathbf{v}_s$, and vehicle horizontal velocity, the apparent grade vertical speed is approximated by

$$v_{\text{grade}} = v_{s,z} - \frac{(\mathbf{V}_{xy} - \mathbf{v}_{s,xy})\cdot\mathbf{n}_{xy}}{\max(0.15, n_z)}.$$

Maximum available upward acceleration is estimated from current vertical thrust, attitude, total mass, and gravity. A bounded descent target grows with square root of height above the ride-height floor. Vertical acceleration feedback uses gain 7 s^{-1} and is converted to collective through the estimated force balance.

Near the ground, attitude reference blends from world-up to the sampled terrain normal over a 4 m height range. Roll feedback uses approximately $I_{\text{roll}}(20e_\phi - 8\omega_x)$. Pitch feedback uses forward-axis alignment with the reference up direction and measured pitch rate, with stronger gains during landing. All requests become front/rear or left/right aperture differences, with explicit saturation handling.

With at least two wheel supports, vertical demand closes. Completion requires a full airborne turn, relative normal speed below 0.5 m/s, angular speed below 0.3 rad/s, alignment with support normal above 0.97, and 0.35 s of stability. There is no independent horizontal-speed completion gate.

Controller success depends on a physically feasible trajectory. It does not guarantee collision avoidance, recovery from every initial momentum, sufficient clearance under overhead structures, or a completed roll for arbitrary custom vehicle/RCS parameters.

14. Terrain, materials, and optional substrate closures

Profiling the sector 16 entry diagnostic retains its vehicle placement and seeded velocity; contact coverage is evaluated in a separate diagnostic run. The independent geometric observer, including its 20 Hz volume probe, is skipped while profiling without changing the applied normal wrench.

14.1 Track definitions and rigid course geometry

The local multiple-track implementation supplies geometry through a process-lifetime immutable `TrackDefinition`. Definitions include identity, limits, base support, route, profiles, sectors, start and recovery points. `CourseTerrain` is instantiated for the active map; its default constructor remains the exhibition for existing laboratory callers. There is no mutable global active route. The environment and vehicle derive their numerical geometry from the same definition, while each terrain instance owns its own topology identity and contact caches.

The `basic-test` map supplies the N3 speed test circuit: approximately 433.33 m inside an 86.41 × 99.52 m design rectangle (8,599.5232 m²), with an approximately 83.8 × 99.3 m road envelope and 8.8 m width. Four main 53 m straights connect through three 12.5 m centerline-radius hairpins and an outer return with two 25 m radius quarter-circles and a 25 m straight. Curves bank inward by up to 6 degrees using 12 m quintic C2 transitions at each end. Level straights and all six recoveries retain a 0.12 m reference top; the banked inner edge stays at or above 0.12 m, while the center reaches approximately 0.5825 m and the outer edge 1.0449 m. Six sectors have no crossings, bridges or obstacles. The infinite analytic apron and concrete column style match the other courses; twelve night floodlights are active. The route assigns approximately 184.53 m to earth and 248.80 m to asphalt, including their bends. N3 uses only `Asphalt=11` with friction/rolling coefficients 0.92/0.018 and rigid compacted-earth `Gravel=1` with 0.68/0.028. The polished low-grip sector has been removed. These assignments preserve the other courses' parameters and add no new surface-force law or deformable terrain.

The independent `obstacle-course` definition adds 16 prescribed rigid sectors with varied surface properties, rollers, steps, a narrow passage, an overpass and a 10 m interruption in an elevated RCS lane. At the gap the road top is 6 m above the foundation, the lane is 4.4 m wide, and the missing span occupies $x \in [-5, 5]$ m around $y = 50$ m. Each solid interval, road edge and deck underside belongs to that definition; the gap is absent from both the visible driving surface and its physical support. The foundation apron remains available at zero height below elevated geometry. The 15 internal safe recoveries exclude the gap. All three courses have an infinite analytic apron. Base support is supplied by the definition's `HasInfiniteApron()` property and must not be inferred from whether a track is the exhibition.

Material identifiers change the existing contact and friction parameters on fixed geometry. They do not create deforming sand or mud, excavation, persistent ruts, Landscape simulation or a new soil constitutive law. The RCS crossing uses the existing force, moment, gas-mass and energy model. Its 10 m geometric span alone does not establish that any vehicle configuration can cross it; local trials must report executable identity, actual initial mass and resources, applied commands and the observed result.

The compact layout retains curved sectors, a mixed-grip lower route and an elevated loop with additional true intersections within an approximately 86.21 × 98.02 m road envelope. Ordinary lanes now use the exhibition course's 4.4 m width; the narrow challenge retains its 3 m lane and approach/departure transitions. Vehicle dimensions, obstacle heights and the 10 m flight gap retain their prescribed scales. Crossing metadata records each lower/upper segment, tangent and support height. Numerical and scene checks retain both layers, test the lower vehicle-clearance prism, and keep the RCS span free of any elevated supporting solid. External rock and road-barrier inventories are reduced to exhibition-like quantities, while curve direction boards and guardrails are absent. Eight ground-level numbered signs remain at sectors 1-7 and 13; boards associated with all raised-road sectors and the tall RCS-gap boards are removed. Illuminated edge markers and the original column style remain. The floating start caption is removed while the start-line markings remain. The four narrow-passage barriers reuse the exhibition body and complete striped base. Their base elevation comes from the apron under the feet rather than the 0.12 m road top, preserving the 3 m body-to-body passage while eliminating visible ground clearance. Elevated T supports reuse a transverse head on one column, with placement checked against lower traffic. Edge-marker yaw follows the mitered road edge, while pitch follows the road's overall grade and roll remains level across the lane. Their height clears local rigid obstacle relief without tilting to individual roller, gravel or block normals. Scenic objects are checked against all road layers and recoveries rather than closing a lane or filling a gap.

Sector 7 uses the 34 m straight after its entrance curve for alternating crossfall. With distance u from the start of that straight and lateral coordinate a, its added height is $E(u)[0.58 + 0.25a\sin(2\pi(u-1)/12)]$ m. The smooth envelope E eases in and out over 3 m and suppresses the profile around the lower bridge crossing, retaining its clear headroom. Maximum transverse slope magnitude is 0.25, or about 14 degrees; the raised center keeps the low side above the apron. These are prescribed rigid heights, not vehicle roll commands or terrain deformation.

Sector 13 preserves its existing mixed wet/dry profile through the first 50% of its route length. The second half contains 33 faceted rock blocks in eleven rows of three, followed by a short rejoining apron-supported road section. A block is 1.36 m wide and 2.62 m long on a 2.8 m longitudinal pitch. Lateral gaps are 0.16 m and longitudinal gaps are 0.18 m. Block rise varies deterministically from 0.14 to 0.20 m above its 0.12 m reference, giving crest heights of 0.26-0.32 m above the apron. Longitudinal bevels are 0.90 m long, with a 0.82 m flat longitudinal crest and shallow side facets. The maximum longitudinal approach rise/run is 0.20. The new tops use the rigid rock surface identifier. Open gaps omit driving-support triangles and have closed exposed block sides; the zero-height foundation remains below. The route-following centerline may continue as a virtual target through a gap, but it supplies no tire support. The same numerical geometry produces visible faces and solid-contact facets. This physical block field is distinct from non-colliding cosmetic photogrammetry scenery.

Native driving trials measure signed progress along the actual route arc, unwrap the closed-loop station coordinate, and reject branch changes that lack corresponding physical movement. Curves do not require a large straight-line displacement between their endpoints. The observer also records vehicle heading changes, grounded-wheel surface identifiers, projected lane clearance and passage through each required intersection at its correct level. Sector-specific drive distances are derived from the route and recovery position; extended lower/upper loop trials continue beyond their final required crossing. These observations do not change forces or freeze the mechanical state. Reported minimum flight clearance is a 20 Hz sampled observation, not a proof for every continuous instant.

Track travel destroys the old world and creates a new vehicle/contact state. In-memory accepted parameters and the remaining RCS air/auxiliary energy are carried as numerical session values. Pose, mechanical velocities, flight valves/controllers and contact caches are reinitialized at the destination start; those discontinuities are imposed initial conditions rather than a continuous mechanical trajectory. The engine

starts running in neutral. Ordinary full-access recovery remains a separate operation with its documented resource reset. Clock and weather preferences are stored by track; the demonstration allowance belongs to the session and does not reset on track travel.

Historical exhibition measurements and validation records refer to their named executable, geometry and test conditions. A successful core regression or a local track-loading check alone does not establish performance or validation of another packaged build.

The exhibition course is prescribed, rigid geometry. Its top is triangulated from the course definition. Tire contact uses geometric faces and actual solid intervals, not interpolated visual shading normals. The course BVH supplies every overlapping solid layer in the tire region. The crest and elevated decks retain their 0.32 m thickness; grounded modules extend to the base plane. The foundation, lower routes, and bridge slabs are combined as a solid union rather than selecting a single highest admissible height. Sector 16 has a 0.45 m baseline and a crest profile reaching 2.05 m.

The spatial index, fine-cell lookup, and conservative height bounds remain available for point-height, compatibility, and ancillary terrain queries. A proof can eliminate a redundant scene ray only where registered geometry and support bounds permit it. Cached point-query cursors belong to one sampling operation and recompute support values at each point. Solid-contact caches retain geometry and prepared quadrature, while recomputing tire overlap and derivatives from the current state.

Immutable terrain adjacency is prepared when the course loads, using the same shared-vertex distance criterion as local topology preparation. Each solid query still derives connected subgroups from its current facet membership, in the same local triangle-pair order. Boundaries belong to those selected subgroups; global connectivity does not supply missing local coverage. Other geometry retains the compatible local preparation path. The plane normal and offset, XY bounds, and canonical edge identities and orientations are also prepared once per triangle. They replace repeated calculations only for valid immutable identities; local selections retain the same vertices, boundaries and insertion order. Nodes retain their XY positions, weights and order; evaluating the stored plane can change rounding in plane coefficients and interval heights.

Topology preparation separates support bodies, which cannot share a connected group. Scene props can therefore use local preparation beside the terrain's immutable adjacency. A body containing incompatible immutable identities retains the generic local path. This separation does not change the ordered interval union across supports.

Immutable terrain facet selection has a separate per-wheel cache keyed by course identity and exact query-XY bounds. Vehicle reset invalidates these selections. The current solver-time scene query is appended afresh, so this cache cannot retain stale scene geometry or rigid support velocities. It is separate from the recent quadrature plans in each preparation cache.

Scene overlap queries use solver-time collision geometry and omit the vehicle itself. Supported scene shapes are converted into projected upper/lower face patches with their solver material and rigid velocity field. The compatibility ray path still requires a sufficiently upward-facing hit above the analytic support: its threshold is $n_z \geq 0.1$, with height tolerances around 1–2 mm depending on the query. Those fallback rules do not control whether a supported solid's upper, side, or lower boundary exists.

The current query still includes all overlapping solid layers, including lower routes beneath elevated decks; optimization does not replace the union by the highest surface. Exact per-cell preparation reuse can avoid rebuilding unchanged intersections when fine query bounds or neighboring candidates change, provided the full reuse conditions remain satisfied. Current scene data are collected before the wheel jobs and remain separate from the immutable facet-selection cache. Dense corrugations, real boundaries and overlapping decks can therefore share preparation without reducing their geometric resolution or the physics update rate.

Scene-detail levels change the rendered meshes and surface shading while retaining the original obstacle collision and tire-support geometry. Photogrammetry rock facets, cosmetic bevels, bolts, support plates, and visual edge wear do not add contact boundaries or modify the coefficients below. Their rendering representation is described in section 15.7.

Course coefficients are:

Surface category	Tire friction μ_s	Rolling coefficient on course tops
Concrete / default rigid top	0.85	0.018
N3 asphalt	0.92	0.018
N3 rigid compacted earth (`Gravel`)	0.68	0.028
Obstacle-course gravel	0.72	0.025
Obstacle-course wet top	0.52	0.018
Polished	0.30	0.010
Steel	0.66	0.018
Rubber	0.96	0.018
Rock	0.84	0.018
Ramp	0.79	0.018
Exterior base concrete plane	1.05	0.016

The exterior analytic base uses 1.05, and the corresponding physical floor component receives a material with dynamic/static friction 1.05 and zero restitution. Track module materials retain the coefficients above. A generic scene support uses the solver physical material's friction when available and a rolling coefficient of 0.016. These are model assignments. A photographed texture, its color, or its apparent wetness does not by itself determine friction.

Obstacles, deck thickness, crossings, ramps, rocks, and barriers alter contact geometry and rigid collision constraints. Their course profiles are designed test geometry, not a simulation of rock fracture, timber flexibility, rubber obstacle deformation, or bridge structural response.

14.2 Legacy soil closure

The core retains sand/mud substrate formulas for compatible test or custom ground providers. None of the three selectable solid-surface tracks instantiates deformable soil patches: each soil collection is empty, stamping is a no-op, and maximum rut depth is zero. The presence of soil functions or surface identifiers does not make the displayed course a persistent terramechanics simulation.

When a compatible provider actually supplies a sand/mud contact, total geometric intrusion is split into carcass deflection and substrate sinkage z_s. A scalar equilibrium solve, using 20 bisection iterations, matches elastic carcass load to an empirical pressure-sinkage support:

$$F_s = A_s K_s z_s^{n_s}, \qquad A_s = w\, 2\sqrt{\max(0, 2R_t d - d^2)},$$

$$d = \min(R_t, z_s + \max(\delta_c, 0)).$$

Mud uses $K_s = 245000$ and $n_s = 1.05$; sand uses $K_s = 720000$ and $n_s = 1.15$, in the implied SI coefficient units. Sinkage is limited to 0.24 m for mud and 0.17 m otherwise. Elastic energy is reduced by the square of the carcass share of intrusion, and sinkage adds rolling resistance.

This is a scalar support closure. It does not resolve soil stress history, shear displacement, compaction, density, moisture transport, bulldozing, track evolution, or a calibrated Bekker/Wong parameter set. In particular, no persistent rut field evolves in the active course.

14.3 Optional water resistance and test terrain

A compatible ground provider can supply water depth. The kernel then adds the longitudinal wheel-center drag

$$\mathbf{F}_w = -280\ \min(d_w, 0.45)\, v_x |v_x|\, \mathbf{e}_x.$$

None of the three active rigid tracks uses this as a fluid domain. The closure has no buoyancy, hydroplaning film, wave dynamics, tire spray momentum, or water transport.

The independent mechanical core also retains a laboratory height/surface provider with fixed triangular sampling. It is useful for native test cases and is separate from the active exhibition geometry. Results from such fixtures should name the actual terrain provider used.

15. Optical, atmospheric, acoustic, and feedback models

These systems affect observation and control feel. Unless explicitly stated otherwise, they do not feed mechanical force, temperature, adhesion, or pneumatic mass back into the vehicle.

15.1 Day/night and scene lighting

The day/night clock is prescribed. For hour H,

$$D = \sin[(H - 6)\pi/12], \qquad \text{sun elevation} = 45D \text{ degrees}.$$

Sun azimuth follows a linear hour relation; intensity is $65000\,\mathrm{clip}(2D, 0, 1)$, with a warmer color at low daylight. Night switching uses $D < 0.04$, with separate moon, sky, stars, and arena-light settings. A cycle of duration t_{day} minutes advances at $\dot{H} = 24/(60 t_{\mathrm{day}})$ hours per real second.

This is an artistic time-of-day construction. It does not compute geographic latitude, date, atmospheric refraction, a solar ephemeris, or an astronomical lunar orbit. Exposure is manually controlled through the scene's time-of-day settings, rather than a physical eye-adaptation model.

The exhibition arena retains all 30 concrete columns and emitter mounting positions, with exactly 15 floodlights enabled in the current night-lighting trial. The obstacle course uses thirty columns and fifteen enabled sources in its own plan; N3 uses twelve concrete columns with twelve enabled sources for its route. The following reference optimization and enable-index mask describe the exhibition plan. The enabled subset keeps its previous aim, intensity, range and color temperature. The full outer beam narrows from 75 to 60 degrees, and the full inner core narrows from 60 to 48 degrees to retain the previous 0.8 core-to-beam ratio and a soft edge. No compensation or re-optimization is applied after selecting the 15 sources or narrowing their beams. Their lens emission is masked per instance; edge-marker emission keeps its original output.

The complete 30-source reference plan at 90 degrees is still reconstructed deterministically to preserve its axes, intensities and ranges. The 60-degree outer beam, 48-degree inner core and 15-source enable mask are applied only after planning. Their final aiming is solved jointly over area-weighted samples across the driving surface, including both steel bands of the twin-track section and the lower and upper bridge levels. Candidate beam axes favor dark sectors over extra light on a bright patch. Five deterministic coordinate passes select axes with target points at least 5 m apart. A separate five-pass output balance reduces excess light in overlapping beams. No aiming optimization runs during gameplay ticks.

The current trial uses a 60-degree full outer beam, a 48-degree full inner core and a 60 m base range. The reference optimizer still uses its original 90-degree full outer beam. After balancing, up to four fixtures may extend to 70 m when their extra reach improves coverage of an insufficiently lit sector. This keeps most shadow volumes local instead of paying for distant overlaps in every fixture. Unreal's current cone half-angles are 30 and 24 degrees; the reference planner retains its original 45-degree outer and 30-degree inner half-angles. Nominal peak intensities before balancing are 120,000 cd for perimeter fixtures and 90,000 cd for interior fixtures; the balance selects output multipliers from 0.25, 0.35, 0.5, 0.65, 0.8 and 1.0. These are simulation lighting parameters, not electrical power ratings or measured luminaire photometry.

At a surface sample with normal n, source-to-sample direction L_i, beam axis a_i, distance d_i and a binary visibility term V_i, the direct-light estimate is

$$E(P) = \sum_i \frac{I_i}{d_i^2} \max(0, -L_i \cdot n) \left[1 - (d_i / r_{\max,i})^4\right]^2 \left[\operatorname{clip}\left(\frac{a_i \cdot L_i - \cos 30^\circ}{\cos 24^\circ - \cos 30^\circ}, 0, 1\right)\right]^2 V_i(P) S_i, \qquad d_i < r_{\max,i} \in \{60, 70\}\ \text{m}.$$

The night-enable multiplier S_i is 1 for fixture indices 0, 1, 4, 5, 8, 10, 11, 14, 16, 18, 20, 22, 24, 27 and 29, and 0 for the other fixtures. The selection preserves light near perimeter curves, obstacles and bridge approaches while allowing darker transitions. The earlier high uniformity/coverage target is retained only as a regression check on the full reference plan; it is not an acceptance criterion for this selective night trial.

Sources outside their range contribute zero. Planning uses two-sided triangle visibility against track surfaces, bridge sides and the concrete columns. This preserves separate bridge levels: a ray below an overhead deck is not incorrectly occluded by the highest surface at the same horizontal position. Runtime diagnostics sample the actual light components and scene collision visibility. Their lux estimates exclude global illumination, material response, tone mapping and differences between visual decoration and its collision proxy; screenshot checks remain necessary. Renderer shadows continue to use the engine's shadow system.

The following cone/plane area relation remains in the legacy mounting-layout reconstruction and geometric regression checks. It does not set the final beam angle or promise a fixed illuminated area. For perpendicular emitter distance h, cone tilt β, and half-angle θ, the footprint ellipse area is

$$A = \frac{\pi h^2 \cos\theta \sin^2\theta}{(\cos^2\theta - \sin^2\beta)^{3/2}},$$

within the bounded-intersection domain. A wide, tilted cone can have an unbounded intersection with an infinite plane; the actual scene and finite attenuation range still bound its visible footprint. The fixed 120 m² footprint used by the earlier independent aiming scheme no longer controls the final lights.

Renderer materials use the engine's lighting, shadowing, reflection, and translucency facilities. Vehicle glazing is a thin-translucent approximation with a configured transmission color, rather than a solved optical solid with measured thickness, dispersion, multiple internal reflections, and structural glass mechanics.

The body finish uses turquoise metallic shading with metallic weight 1, base roughness 0.105, clear-coat weight 1, and coat roughness 0.055. A small procedural normal perturbation provides the microfinish; rubber, trim, and mirrors use separate finishes. These are artistic shader parameters, not measured spectral reflectance, material strength, collision restitution, or tire adhesion.

The renderer is configured for Lumen hardware ray tracing with lighting evaluated at specular reflection hits, up to two reflection bounces, front-layer glass reflections, and sun contact shadows. Indirect diffuse lighting retains its cache. These are rendering approximations whose availability and cost depend on the active renderer and GPU; the polished image does not change the vehicle's mass, force laws, contact geometry, or mechanical integration.

15.2 Global fog and wind-driven dust

Global fog is the engine's exponential-height/volumetric fog representation. Controls set density, height, falloff, scattering tint, and renderer quality. The tint interpolates between cool and warm albedos. Fog has zero self-emission and a configured scattering asymmetry of 0.2.

Dust consists of a bounded pool of six gusts with four local fog lobes per gust. Each gust is spawned near the vehicle's area, anchored to sampled terrain, and then advected from a fixed world origin:

$$\mathbf{x}_{\text{gust}}(t) = \mathbf{x}_0 + \mathbf{e}_{\text{wind}} v_{\text{wind}} t.$$

Let $s = t/t_{\text{life}}$ and $S(x) = \bar{x}^2(3 - 2\bar{x})$, with $\bar{x} = \text{clip}(x, 0, 1)$. The life envelope is

$$e(s) = S(s/0.18)\, S((1 - s)/0.36).$$

Lobes expand with age and receive sinusoidal offsets and density modulation. Local scattering has a warm dust albedo, zero emission, and asymmetry 0.25. Randomized size, duration, phase, and spawn interval give visual variation.

No Navier–Stokes solver, particle mass conservation, wind-pressure force, dust deposition, tire abrasion source, or visibility-dependent tire friction is present. Fog and dust density sliders are rendering controls, not calibrated atmospheric aerosol concentrations.

Wheel-contact dust is a separate, bounded visual source. It reads published wheel-contact state without writing forces, soil state, or contact geometry. Sand, gravel, and dry-rock categories emit when the wheel is supported, normal load is at least 100 N, the contact normal has a vertical component of at least 0.25, and vehicle/wheel motion exceeds the emission threshold of 0.6 m/s. Density scales with the independent wheel-dust intensity, a category factor, motion, and slip. Wet, mud, steel, and clean-pavement categories do not emit this dry dust.

Wheel clouds update at approximately 30 Hz. Performance / Balanced / High weather quality limits the pool to 8 / 16 / 24 active clouds and uses per-wheel emission intervals of 0.65 / 0.45 / 0.35 s. A long rendering frame caps the visual update increment at 0.1 s instead of producing catch-up emissions. Each cloud has a prescribed lifetime of 1.6–2.8 s, an initial radius of 0.18–0.28 m, and radial growth of 0.24 m/s. Using the smooth-step function S defined above, its extinction envelope is

$$e_{\text{wheel}}(s) = S(s/0.15)(1 - s)^2, \qquad s = t/t_{\text{life}}, \quad 0 \le s \le 1.$$

Clouds remain in world space, move with a prescribed fraction of visual wind and turbulence, and rise relative to the original support plane, including on elevated decks. Local fog volumes scatter scene lighting with zero self-emission; their extinction settings are not a mass-conserving aerosol model. Wheel dust has independent enable/intensity controls and shares weather quality with global fog and gusts. The shared

volumetric renderer is disabled when fog and gusts are inactive and no wheel clouds remain. This presentation does not consume RCS gas or change tire wear, adhesion, suspension, or vehicle aerodynamic drag.

15.3 Visible RCS exhaust and ground interaction

Visible exhaust follows the actual post-pressure/post-flow nozzle force. Jet length scales with the square root of the nozzle's force fraction; ray queries shorten it against the scene. Lower quality uses fewer probes and fewer rendering samples.

Airborne and ground-interaction clouds use a fixed pool with finite lifetimes. Their initial velocity combines vehicle point velocity, a prescribed exhaust/radial velocity, and a fraction of visual wind. Velocities decay exponentially; radius grows with age. Surface traces clip or redirect cloud motion near impact surfaces.

A cloud's density envelope includes smooth birth, an age falloff approximately $(1 - t/t_{\text{life}})^{1.6}$, and a radius dilution factor proportional to $(R_0/R)^{0.65}$. This creates an interpretable visual plume/condensation effect. It is not a gas mass-density solution, a prediction of condensation chemistry, or a resolved impinging jet. Visual ground interaction adds no extra thrust, ground-effect lift, or collision force to the vehicle.

At runtime, Low / Balanced / High quality ray-marches the procedural volume with 16 / 28 / 32 samples. For each sample optical depth $\Delta\tau$,

$$\alpha = 1 - e^{-\Delta\tau}, \qquad \mathbf{L}^{+} = \mathbf{L} + \mathcal{T}\,\alpha\,\mathbf{L}_{\text{incident,scatter}}, \qquad \mathcal{T}^{+} = \mathcal{T}(1-\alpha).$$

Opacity is $1 - \mathcal{T}$. Density, scene-depth clipping, and user appearance controls determine optical depth. Its scale is a rendering closure, not a conversion from the tank's kilograms to optical extinction.

Incident illumination is sampled from actual scene lights and visibility, including up to two directional and eight selected local sources per volume. Local attenuation uses inverse square distance with a smooth finite-range term; spotlights include cone falloff. Directional atmospheric transmission and sky visibility are approximated. The phase function is Henyey–Greenstein-like with $g = 0.2$:

$$p(\cos\vartheta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\cos\vartheta)^{3/2}}.$$

The material integrates single scattering with a near-neutral tint. There is no independent luminous exhaust source. Its unlit-material emission output carries computed outgoing scattered radiance; that implementation channel does not represent combustion or self-emission. Multiple scattering and fully resolved volumetric light transport are not calculated by this custom plume model.

15.4 Procedural sound

Engine audio is synthesized from a fundamental $f = n/30$ Hz, harmonics at $2f$ and $4f$ with relative amplitudes 0.4 and 0.2, and filtered pseudorandom noise. Running amplitude depends on normalized engine load. Frequency and amplitude use per-audio-sample smoothing.

RCS audio is a filtered-noise contribution whose level follows normalized total actual nozzle thrust. Despite an internal “flow” name, this audio driver is not the physical kg/s flow measurement. The synthesis is a perceptual cue; it does not solve exhaust acoustics, cylinder firing pressure waves, structural radiation, Doppler propagation, or a calibrated sound-pressure level.

15.5 Camera

The external camera pivot tracks the physical chassis center of mass. Its heading frame uses the horizontal projection of the forward direction with continuous angle unwrapping, retaining the last heading near vertical orientation. Roll and pitch of the body do not rotate the external yaw frame as a cabin camera would.

The obstruction probe shortens camera distance immediately and restores it exponentially with an approximately 0.25 s time constant. The center-of-mass anchor itself is refreshed directly; the camera is not a physical tethered mass or a suspension sensor. Cabin view follows the full body pose. Camera motion has no feedback into vehicle dynamics.

15.6 Haptics and input transfer

Pedals and steering axes are calibrated by piecewise linear center/end-point mappings, dead-zone removal, inversion, and configurable pedal response exponents. These mappings are part of the control transfer chain. They are not engine, brake, or rack constitutive laws. A missing required wheel/pedal device selects a neutralized input with brake demand, rather than inventing a physical hardware state.

Wheel force feedback uses front lateral force, steering-wheel ratio, compression-rate difference, and input wheel rate:

$$u_{\text{align}} \approx -\frac{0.07 \sum_F F_y}{\text{clip}(r_s, 2, 40)\, 8}, \qquad u_{\text{impact}} = 0.025(\dot{c}_{FR} - \dot{c}_{FL}).$$

The normalized target combines these with $-0.04\dot{\theta}_{\text{wheel}}$, applies configured gain/inversion, and limits slew. The 0.07 m effective trail and 8 N m reference are feedback-design constants, not a calibrated steering-rack or device motor model. Loss of supported front contact or device capability clears torque.

Controller vibration combines weighted absolute slip, summed compression rates, and ABS activity into two bounded motor amplitudes. These cues do not inject a reaction torque or energy from the user's physical controller into the simulated chassis.

15.7 Rendering and visual scene detail

Anti-aliasing and upscaling reconstruct the displayed image independently of the mechanical solver. Native TSR and NVIDIA DLAA use native input resolution. DLSS Quality / Balanced / Performance use the input scale supplied by the NVIDIA SDK for the current output size. Automatic requests DLAA at up to 2,073,600 output pixels and DLSS Quality above that count; an unavailable NVIDIA mode falls back to Native TSR. Resizing the viewport reevaluates the active mode and input size. Dynamic resolution, NVIDIA frame generation, and Ray Reconstruction are disabled by this implementation. Reducing input resolution can lower GPU cost without improving a CPU-limited simulation's frame rate.

Detailed and Full scene levels replace selected renderers with instanced Nanite meshes, including beveled fixtures and three Electric Dreams / Quixel Megascans rock variants with PBR textures. Full also enables fasteners and support plates. These cosmetic components have collision disabled; original physical obstacles remain active even when their simpler renderers are hidden. Essential uses the simpler fixture geometry while retaining the corrected structural finishes. The fine photographed rock relief and visible bevels therefore do not define additional tire support or rigid-body collision surfaces.

The apron blends three deterministically offset samples of the existing concrete maps using continuous triangular weights. Explicit derivatives of the unshifted world-space UVs select mip levels independently of the random offsets. Broad variation acts at multiple spatial scales and fades when its features are smaller

than a pixel's ground footprint. Photographic detail and normal perturbations fade toward the source image's linear-light average and a flat normal over 40–220 m viewing distance. This reduces visible tiling and unresolved distant texture without changing the support plane or its friction.

Bridge members, sign faces, and signposts have separate roughness/metallic finishes without plate-border photographs or strong normal-map bands. Cosmetic longitudinal bridge beams are narrowed by 2 cm on each side so their outer faces no longer coincide with the matte-black deck sides. Their original collision instances and the bridge's physical deck geometry remain unchanged at every detail level.

Column concrete uses continuous world-space triplanar texture projection across the column and base. Beacon stripes use wider bands and screen-derivative filtering that blends unresolved stripes toward their average at distance. Surface variation, roughness, normal maps, and apparent edge wear affect light scattering rather than soil state, friction coefficients, or the physical tire-wear variable. Anti-aliasing and scene detail persist together in Graphics.json; changing them leaves the selected mechanical parameters and physics update rules intact.

16. Observables and energy accounting

16.1 Acceleration display

Motion telemetry differentiates world center-of-mass velocity:

$$\mathbf{a}_{\mathrm{raw}} = (\mathbf{V}_k - \mathbf{V}_{k-1})/\Delta t.$$

It filters in world coordinates with a 0.045 s time constant, then projects into the current body frame. Differentiating already rotated local velocity would introduce a different rotational term; that is not how this display is formed.

The display is kinematic acceleration, not accelerometer specific force $\mathbf{a} - \mathbf{g}$. A resting vehicle reads approximately zero. A freely falling vehicle has world vertical acceleration near $-g$, subject to filtering and other forces. Local signed peaks retain the strongest absolute event for a short interval. Invalid intervals and reset/recovery transitions reset history to avoid treating a teleport as measured acceleration.

RCS bank percentages are based on actual front-plus-rear forces divided by the two-nozzle nominal bank force. Pressure loss, valve response, and shared-flow saturation can therefore make actual percentages differ from requested aperture.

16.2 Mechanical energy observer

The diagnostic mechanical energy is the sum of:

1. Sprung center-of-mass translational energy, full-tensor rotational energy, and gravitational potential.
2. Both axle assemblies' translational, rotational, and gravitational energy, evaluated from the reduced geometry and velocities.
3. Four wheel spin energies and engine rotor energy.
4. Spring, anti-roll, travel-stop, and stored tire elastic potential.

In compact form,

$$E_{\mathrm{mech}} = \tfrac{1}{2}M|\mathbf{V}|^2 + \tfrac{1}{2}\boldsymbol{\omega}^{\mathsf{T}}\mathbf{I}_C\boldsymbol{\omega} + Mgz_C + \tfrac{1}{2}I_e\omega_e^2 + \sum_a (E_{a,\mathrm{kinetic}} + m_a g z_a) + \sum_i \left(\tfrac{1}{2}I_w\omega_i^2 + U_{t,i} + U_{s,i} + U_{\mathrm{stop},i}\right) + \sum_a U_{\mathrm{bar},a}.$$

The tire contribution uses the stored contact-potential value, including its large-penetration continuation, rather than assuming every contact energy is $\frac{1}{2}k_t\delta_c^2$. Suspension energies use seat distances consistent with the force model.

This observer does not alter forces, clamp trajectories, or feed back into the solver. It is not an energy correction mechanism.

16.3 What an energy residual means

The mechanical observer excludes thermal energy inventories, battery energy, gas thermodynamic exergy, fuel chemical energy, the external environment's gravitational/contact energy, and a resolved exhaust field. Tire/brake heat equations are aggregate closures and do not establish a complete energy ledger for every modeled loss.

For a controlled test, a residual can be defined as

$$\mathcal{R}(t) = E_{\mathrm{mech}}(t) - E_{\mathrm{mech}}(0) - \int_0^t \left(P_{\mathrm{supplied}} - P_{\mathrm{dissipated}}\right) dt.$$

It is meaningful only after specifying which powers are included and holding model settings and mass-resource operations consistent. A decline in mechanical energy with active damping, braking, rolling loss, drag, or inelastic contact is expected. An increase during engine or RCS operation is not automatically numerical energy creation.

Potential sources of unexplained residual include partitioned chassis coupling, filtered base acceleration, finite quadrature and support transitions, travel/force projection, finite-capacity clutch and differential approximations, rotor-axis simplifications, native collision stabilization, and mass-property update lag. Parameter edits, recovery, gas refill, and moving supports supply or remove energy externally.

An accurate gradient for one elastic contact calculation does not establish exact global energy or momentum conservation for the complete hybrid simulator.

17. Interpreting experiments and model limits

17.1 Reproducible comparisons

Record vehicle dimensions, mass distribution, per-axle suspension settings, individual cold pressures, tire/brake initial temperatures, transmission and differential modes including the selected 4x2 axle, active assistance, RCS equipment/resources, terrain location/material, and the actual input source. Record whether fast refill, ABS, hill assistance, RCS damping, and automatic control are enabled.

Start comparison runs from the same initialized state and support geometry. Do not treat a recovery, structural settings edit, or resource reset as ordinary continuous evolution. Distinguish the configured fixed physics interval from achieved render frame rate; visual quality is not a tire integration-rate control.

The following checks help interpret the equations:

Experiment	Expected model relationship	Main qualification
Static flat support	Sum of tire loads approaches total vehicle weight	Body protective shapes must not carry part of the load

Experiment	Expected model relationship	Main qualification
Small symmetric axle motion	Spring and damping response follows seat geometry	Seat motion ratio and tire stiffness prevent a simple wheel-rate substitution
Articulation with bar on/off	Differential compression changes roll resistance	End stops, load transfer, and linkage motion also contribute
Tire pressure sweep	Pressure changes vertical/slip stiffness, radius estimate, rolling loss, and heat response	No measured tire pressure/load map is fitted
Open versus locked axle	Lock resists wheel-speed difference with bounded transfer torque	It does not impose exactly equal speeds
Rear versus Front 4x2	Engine input and shaft-speed coupling follow the selected axle	Hold differential modes, mass, support, gear and inputs constant; non-driven axle differential reactions can remain nonzero
Coast-down	Drag scales approximately with speed squared, with rolling/drivetrain losses	Gear, clutch state, grade, brakes, and wind-independent drag must be controlled
Brake repetition	Temperature reduces service torque through fade	ABS and load distribution can dominate the measured stop
RCS impulse budget	Consumed gas equals summed scalar nozzle impulse divided by v_e	Net vector impulse can be smaller because nozzle forces cancel
RCS tank depletion	Pressure is linear in usable gas; thrust weakens below regulator pressure	Shared flow and valve history also limit force
Compressor refill	Refill rate decreases as isothermal work per kilogram rises	Fast recharge must be disabled
Free-fall telemetry	Kinematic acceleration includes gravity	The display is not specific force

These are interpretation targets, not claims that a new runtime validation campaign has been performed for every parameter combination.

17.2 Useful linearized estimates

For small near-vertical motions and a fixed chassis, two equal per-side springs give an axle heave stiffness near $2k$ before linkage motion ratios. A nominal uncoupled frequency estimate is $f_n \approx (2\pi)^{-1}\sqrt{2k/m_a}$. It is not the full sprung-body ride frequency.

For an isolated per-side vertical tire and suspension spring in series, $k_{\text{eff}} \approx (k_s^{-1} + k_t^{-1})^{-1}$ is a first estimate. The actual seat motion ratio, solid-axle roll, anti-roll bar, tire load sensitivity, and body coupling change the modes.

For RCS with constant net force direction, slowly varying mass, and no saturation change, $\dot{\mathbf{V}} \approx \mathbf{F}/M + \mathbf{g}$ is a local estimate. Over a substantial gas discharge, use instantaneous mass and actual force; a single full-tank acceleration is insufficient.

Such estimates are useful for orders of magnitude and comparison design. They should not replace the coupled nonlinear equations near contact loss, travel stops, high slip, saturation, or a full roll.

17.3 Applicability boundary

The simulator supports physical experimentation with its implemented generic vehicle and controls. Predicting a particular real vehicle would require measured mass/inertia, suspension hard points and compliance, tire data, drivetrain maps, brake thermal/friction data, and RCS hardware characterization.

High-speed crashes, tire rupture, thermal destruction, powertrain failure, real-gas high-pressure discharge, compressor thermal limits, deformable soil trafficability, buoyancy, hydroplaning, and resolved atmospheric aerodynamics lie outside the implemented model. Numeric parameter bounds protect configuration validity; they are not certified operating envelopes or validated hardware ratings.

Source-level consistency, native tests, packaged execution, measured physical calibration, and validation against independent experiments are different evidence levels. This document describes the implementation and its assumptions; it does not substitute for any of those validation activities.

18. Default parameters and implementation map

18.1 Mechanical and pneumatic reference values

Quantity	Default	Meaning
Base sprung mass	1,850 kg	Excludes optional loads, RCS, and axle assemblies
Unsprung mass parameter	47.5 kg per wheel	95 kg per solid axle
Wheelbase / track	2.90 / 1.68 m	Nominal axle spacing and wheel-center separation
Tire radius / width	0.435 / 0.265 m	Undeformed geometric parameters
Wheel spin inertia	2.7 kg m²	Each wheel
Front/rear spring rate	43,000 N/m per side	Spring-seat force law
Compression / rebound damping	2,800 / 4,200 N s/m	Digressive above 0.45 m/s seat speed
Front/rear reference length	0.46 m	Axle/spring reference geometry
Compression / droop travel	0.23 / 0.12 m	End-coordinate travel envelope
Front/rear anti-roll coefficient	9,000 N/m	Differential seat compression
Front/rear axle roll inertia	55 kg m²	Reduced axle inertia parameter
Front/rear arm length	0.95 m	Fixed-length linkage constraints
Tire vertical stiffness / damping	200,000 N/m / 350 N s/m	Before pressure scaling
Tire longitudinal / lateral stiffness	90,000 / 78,000 N	Before pressure scaling
Tire relaxation length	0.22 m	With a 2 m/s relaxation-speed floor
Cold gauge tire pressure	2.2 bar	Independently adjustable per tire
Engine peak torque / inertia	360 N m / 0.32 kg m²	Map scale and lumped rotor
Engine idle / maximum speed	850 / 5,800 RPM	Idle governor and torque cut
Clutch capacity / drive efficiency	650 N m / 0.92	Aggregate coupling and power loss

Quantity	Default	Meaning
Service / rear parking reference torque	2,600 / 2,200 N m	Per-wheel torque-law scales
Base front brake fraction	0.64	Before load-sensitive EBD
Brake heat capacity / base cooling	18,000 J/K / 35 W/K	Cooling also grows with speed
Ambient temperature	20°C	Tire/brake reference and compressor intake temperature
Drag area / air density	0.95 m² / 1.225 kg/m³	Stationary ambient aerodynamic drag
Steering angle / rate / response	34° / 1.8 rad/s / 0.10 s	Non-direct steering actuator
RCS usable gas / dry mass	100 / 80 kg	Residual gas adds further mass
RCS vertical / lateral nozzle force	10,000 / 2,000 N	Each nozzle at full authority
RCS equivalent exhaust speed	500 m/s	Consumption/impulse closure
RCS maximum shared flow	100 kg/s	Shared by all eight nozzles
RCS valve opening response	0.025 s	Immediate closure at zero request
RCS full / cutoff / regulator pressure	300 / 20 / 40 bar absolute	Isothermal storage and authority
RCS reserve	25% of usable capacity	Ordinary lateral-demand policy
Compressor power / efficiency	30 kW / 0.5	Physical refill mode
Auxiliary battery energy	30 kWh	Finite compressor energy inventory
Fast recharge time	5 s	Virtual refill; enabled by default

Initial modes include automatic transmission, clutch assistance, ABS, and hill-start assistance enabled; 4WD, low range, axle locks, limited-slip options, and hill descent are initially off. The default 4x2 driven axle is Rear; Front is selectable while 4WD is disabled. RCS equipment is initially not installed. Saved configurations take precedence over these defaults. The case-sensitive `TwoWheelDriveAxle` setting accepts `Rear` or `Front`; an absent, unknown, or incorrectly typed field uses Rear.

18.2 Implementation map

The following paths identify the source implementation for advanced readers who have the development project. Source files are not required to run the simulator and are not included in the public runtime packages.

Source path, relative to the project root	Responsibility
Source/OffroadSimulator/VehicleDynamics.h	Parameters, units, core state, pose and ground interfaces
Source/OffroadSimulator/VehicleDynamics.cpp	Substeps, private wheel-contact batches and ordered consumption, engine/transmission, differentials, wheel spin, traction, tire temperature/wear, drag

Source path, relative to the project root	Responsibility
Source/OffroadSimulator/SuspensionModel.cpp	Mass tensor, linkage closure/Jacobians, reduced axle equations and reactions
Source/OffroadSimulator/RoundedTireVolume.h	Rounded carcass weighting, line/plane integrals and inverse depth calibration
Source/OffroadSimulator/TireContact.h	Compatibility height-sampler potential, contact derivatives, normal wrench and support shares
Source/OffroadSimulator/SolidTireContact.h	Solid-interval union, clipped-cell quadrature, plane consistency, per-cell reuse eligibility and current support data
Source/OffroadSimulator/SolidTireWindow.h	Per-wheel fine queries, proved terrain windows, retained selections and shared cell-preparation memo
Source/OffroadSimulator/SolidTireCellPreparation.h	Bounded exact-key storage of per-cell nodes and intervals, admission and retirement
Source/OffroadSimulator/SolidTireBoundaryHalo.h	Bounded boundary-halo grid with the direct predicate retained as fallback
Source/OffroadSimulator/SolidTirePreparationStorage.h	Complete preparation-plan storage and reusable working buffers
Source/OffroadSimulator/OffroadSceneSolidGeometry.h	Solver-time box, convex and closed-mesh collision faces, projected patches and affine lower planes
Source/OffroadSimulator/BrakeSystem.h	Pressure response, EBD, ABS, thermal fade, hill hold and descent
Source/OffroadSimulator/SteeringResponse.h	Input response curve and critically damped steering actuator
Source/OffroadSimulator/DrivingControls.h	Calibration, control resolution, normalized force feedback and rumble
Source/OffroadSimulator/OffroadVehicle.cpp	Physics-thread coupling, SI conversion, body loads and runtime mass update
Source/OffroadSimulator/OffroadVehicleContact.cpp	Serial solver-time terrain/scene snapshots, eligible parallel contact dispatch and dynamic-support reactions
Source/OffroadSimulator/OffroadModernBody.cpp	Fitted visible body geometry
Source/OffroadSimulator/OffroadVehicleCollision.cpp	Welded collision shapes, physical material and native body setup
Source/OffroadSimulator/VehicleBodyFit.h	Shared body/nozzle dimensional fitting
Source/OffroadSimulator/RcsDynamics.h and RcsDynamics.cpp	Gas storage, regulation, valves, thrust, recharge, mass and intake momentum
Source/OffroadSimulator/RcsPitchAllocation.h	Bounded roll-preserving pitch allocation
Source/OffroadSimulator/RcsAutoRoll.h	Automatic roll trajectory phases, braking and terrain-relative landing

Source path, relative to the project root	Responsibility
Source/OffroadSimulator/TrackDefinition.h	Immutable per-track routes, limits, profiles, starts and recoveries
Source/OffroadSimulator/CourseTerrain.h and CourseTerrain.cpp	Per-track triangles/materials, topology identity, support indexing and retained substrate closures
Source/OffroadSimulator/OffroadSession.h and OffroadSession.cpp	Numerical session transfer between maps; resources are retained while mechanical initial conditions restart
Source/OffroadSimulator/MechanicalEnergy.h	Passive mechanical energy observer
Source/OffroadSimulator/MotionTelemetry.h	Kinematic acceleration filtering and actual RCS bank indicators
Source/OffroadSimulator/OffroadEnvironment.cpp	Scene geometry, day/night lighting and arena calibration
Source/OffroadSimulator/ArenaLighting.h and ArenaLighting.cpp	Cone footprint and illuminance placement model
Source/OffroadSimulator/OffroadWeather.cpp	Global fog, gust lifetimes, dust advection and rendering parameters
Source/OffroadSimulator/OffroadWheelDust.cpp	Read-only wheel-contact dust source, bounded cloud pool and visual advection
Source/OffroadSimulator/OffroadGraphicsSettings.cpp	Anti-aliasing/upscaling selection, resolution policy and independent graphics persistence
Source/OffroadSimulator/OffroadSceneDetails.cpp	Instanced Nanite presentation, detail visibility and preservation of original collisions
scripts/GenerateModernVehicleMaterials.py	Polished metallic body, clear coat, trim, mirrors and rubber shaders
scripts/SceneSurfaceMaterials.py	Continuous column concrete, filtered beacon stripes, stochastic ground sampling and separate structural finishes
Source/OffroadSimulator/OffroadRcsVisuals.cpp	Force-driven plume/cloud geometry and visual ground interaction
Source/OffroadSimulator/OffroadRcsScattering.cpp	Scene light sampling and plume illumination/visibility approximation
scripts/GenerateRcsMaterials.py	Procedural exhaust density and single-scattering ray integration
Source/OffroadSimulator/OffroadEngineSound.cpp	Procedural engine and RCS audio synthesis
Source/OffroadSimulator/ChaseCameraFilter.h	Center-of-mass camera frame and obstruction-distance response
Config/DefaultEngine.ini	Requested asynchronous physics interval, gravity and renderer configuration

For the configuration controls that expose these parameters and the complete operating procedures, use the User manual. Keep both documents together in the distribution's documents folder.